Post-Quantum Adaptor Signature for Privacy-Preserving Off-Chain Payments

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Introduction and Motivation

Adaptor Signatures

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- Recently formalized by Aumayr et al. in [AEE+20]

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- Applications: payment channel networks, payment channel hubs, atomic swaps, etc.

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  - e.g., knowledge of discrete logarithm relation and Schnorr/ECDSA signature
- Can be viewed as a **two party "conditional signature"**
  - But, the final output is an ordinary signature from a single party (or a joint signature in the threshold setting)

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  - $y/\perp \leftarrow \mathsf{Ext}(\sigma, \hat{\sigma}, Y)$ such that $(Y, y) \in R$

Adaptor Signature - Protocol View

$(Y, y) \in R$
pk, m

$(\mathsf{sk}, \mathsf{pk})$
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$y \leftarrow \text{Ext}(\sigma, \hat{\sigma}, Y)$

Observation:

- PreSig is like a **commitment**, such that Alice with a valid witness can **complete** the signature. And any valid $(\sigma, \hat{\sigma})$ pair **reveals** a witness.

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- **Witness Extractability:** any given (pre-signature, signature) pair on the same statement Y reveals a witness to Y

Instantiation and Application

Schnorr Signature

Public parameters: group parameters $(\mathbb{G}, g, p)$, hash function $H\colon \{0,1\}^* \to \mathbb{Z}_p$

procedure KGen(1^λ)

$x \leftarrow_\$ \mathbb{Z}_p$

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$e := H(\mathsf{pk} \| R \| m)$
$s := k + e \cdot \mathsf{sk}$
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procedure Ver(pk, m, σ)
Parse σ as (e, s)
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procedure Adapt($\hat{\sigma}$, y)
- Parse $\hat{\sigma}$ as $(e, \hat{s})$
- $s := \hat{s} + y$
- **return** $\sigma := (e, s)$

procedure Ext(σ, $\hat{\sigma}$, Y)
- Parse σ as (e, s), $\hat{\sigma}$ as $(e, \hat{s})$
- $y' := s - \hat{s}$
- **if** $(Y, y') \in R$ **return** y'
- **else return** $\perp$

Payment Channel Networks (PCNs)

- Malavolta et al. [MMS+19] introduced Anonymous Multi-Hop Locks (AMHLs) as a building block for secure and privacy-preserving PCNs

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$y_1 \xleftarrow{\$} \mathbb{Z}_p$; $Y_1 := g^{y_1}$

for $i \in [2,4]$: $y_i \xleftarrow{\$} \mathbb{Z}_p$

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A

E_1

B

E_2

C

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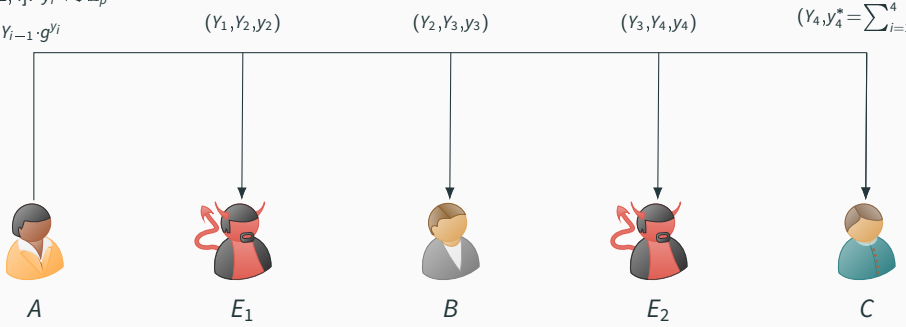

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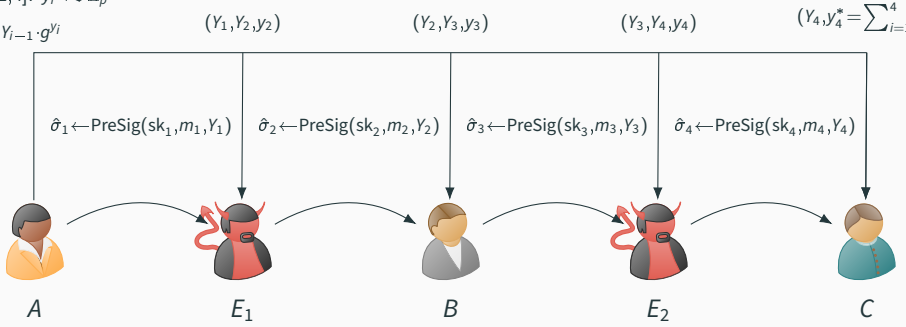

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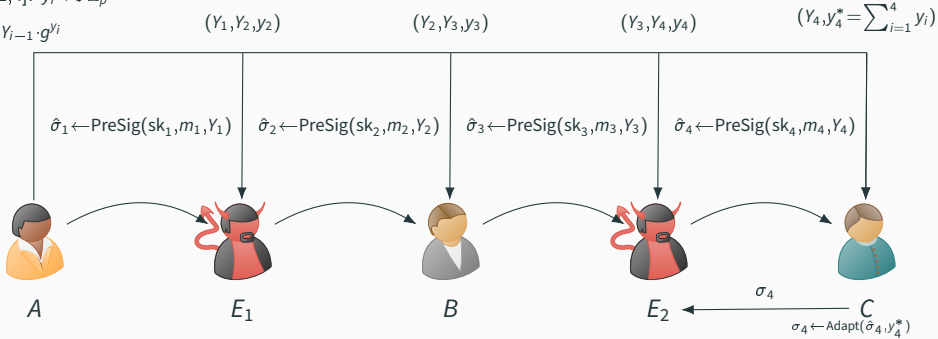

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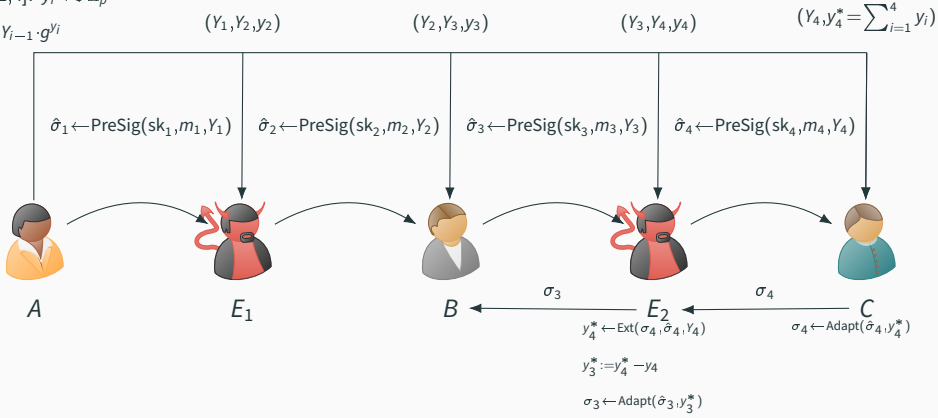

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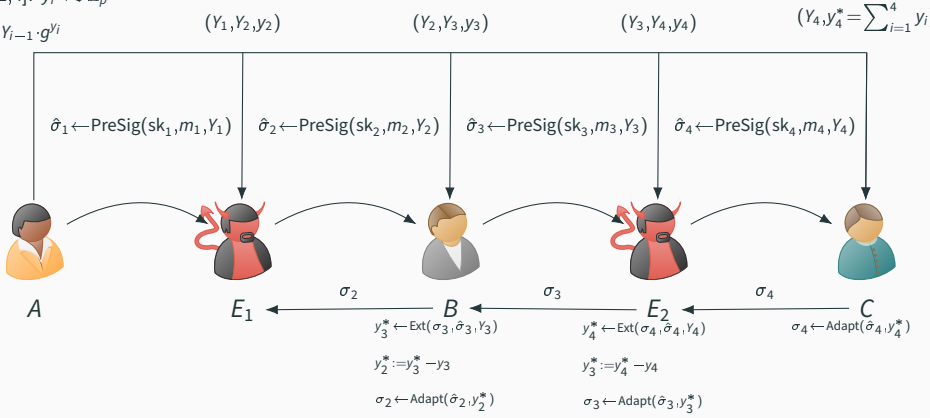

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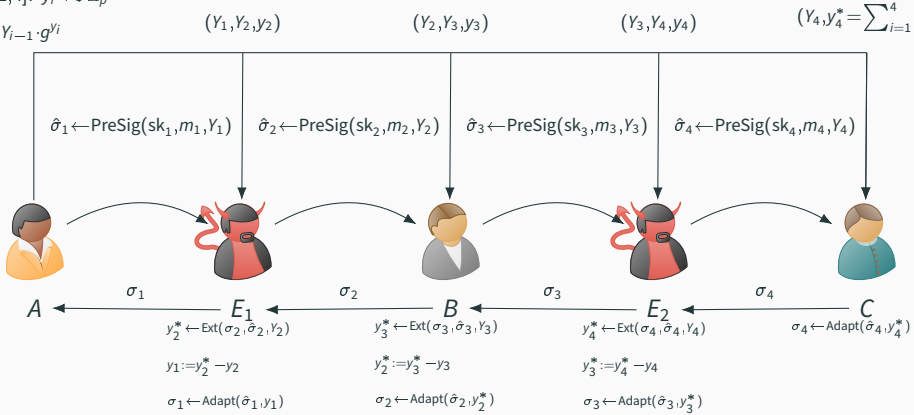

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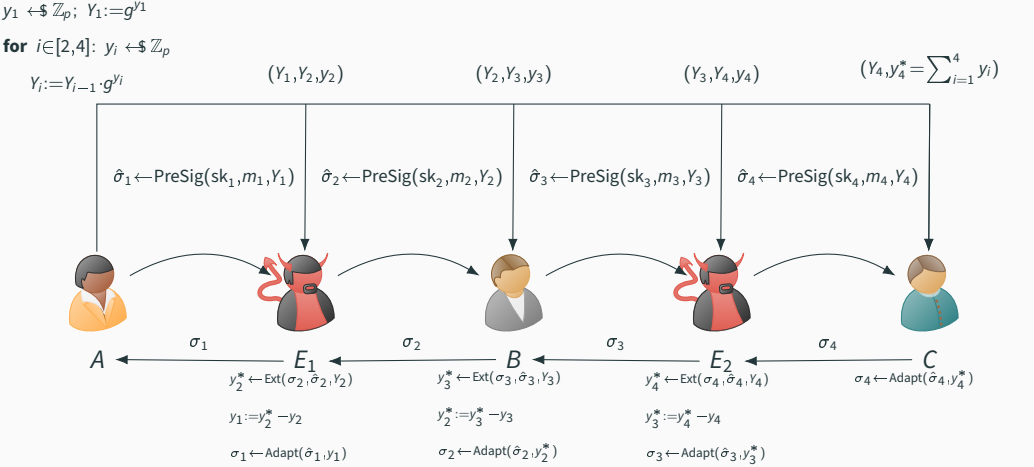

- Additionally, the sender (A) needs to send a ZK proof for the well-formedness of objects to each intermediary.

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Can we construct adaptor signatures that remain secure against quantum adversaries?

Yes!

Lattice-based Adaptor Signature

Lattice Adaptor Signature (LAS)

- Introduced by Esgin et al. [EEE20]
- Builds upon the Dilithium signature scheme [DKL$^+$18], which uses Fiat-Shamir with Aborts technique (rejection sampling)
- Based on well-known Module-SIS and Module-LWE problems
- Hard relation: knowledge of Module-SIS preimage
- Post-quantum PCNs and atomic swaps using LAS

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procedure KGen(1^λ)

$\mathbf{x} \leftarrow_\$ \mathbb{S}_1^{n+\ell}$

return $(\mathsf{sk} := \mathbf{x}, \mathsf{pk} := \mathbf{Ax})$

- $\mathbb{S}_1^{n+\ell}$ is a distribution that samples vectors of size $n + \ell$ and of norm 1

Schnorr

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$e := H(\mathsf{pk} \| R \cdot Y \| m)$

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procedure PreSig(sk, m, $\mathbf{Y}$)

$\mathbf{k} \leftarrow\!\!\$\ \mathbb{S}_\gamma^{n+\ell}, \mathbf{R} := \mathbf{Ak}$

$e := H(\mathsf{pk} \| \mathbf{R} + \mathbf{Y} \| m)$

$\hat{\mathbf{s}} := \mathbf{k} + e \cdot \mathsf{sk}$

if $\|\hat{\mathbf{s}}\|_\infty > \gamma - \kappa - 1$ **then**

Restart

return $\hat{\sigma} := (e, \hat{\mathbf{s}})$

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- Range of H (challenge set $\mathcal{C}$) is composed of elements from the ring $\mathcal{R}$ with certain norm restrictions
- If statement is used to perform rejection sampling, in order not to leak information about the secret key

Schnorr

procedure PreVer($\mathsf{pk}, m, Y, \hat{\sigma}$)

Parse $\hat{\sigma}$ as $(e, \hat{s})$

$e' = H(\mathsf{pk} \| g^{\hat{s}} \cdot \mathsf{pk}^{-e} \cdot Y \| m)$

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  if $\|\hat{\mathbf{s}}\|_\infty > \gamma - \kappa - 1$ **then**
    return 0
  $\mathbf{w} := \mathbf{A}\hat{\mathbf{s}} - e \cdot \mathsf{pk}$
  $e' = H(\mathsf{pk} \| \mathbf{w} + \mathbf{Y} \| m)$
  return $(e = e')$

LAS

- Adapt and Ext algorithms are analogous to Schnorr-based construction.
- The hard relation R considered so far is knowledge of Module-SIS with a small preimage (i.e., $(\mathbf{Y}, \mathbf{y}) \in R$ iff $\mathbf{Y} = \mathbf{A}\mathbf{y}$ and $\|\mathbf{y}\|_\infty \leq 1$)

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procedure Ext(σ, $\hat{\sigma}$, Y)
  Parse σ as $(e, \mathbf{s})$, $\hat{\sigma}$ as $(e, \hat{\mathbf{s}})$
  $\mathbf{y}' := \mathbf{s} - \hat{\mathbf{s}}$
  if $(\mathbf{Y}, \mathbf{y}') \in R$ **return** $\mathbf{y}'$
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Observation:
The witness $\mathbf{y}'$ output by Ext can have norm at most $\leq 2(\gamma - \kappa)$.

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  - $\sigma \leftarrow \text{Adapt}(\hat{\sigma}, y)$ where $(Y, y) \in R$
- **Witness Extractability:** any given (pre-signature, signature) pair on the same statement Y reveals a witness y' such that $(Y, y') \in R'$
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Solution:
- Use a lengthy ZK proof (e.g., the proof from [ENS20] is 47KB).

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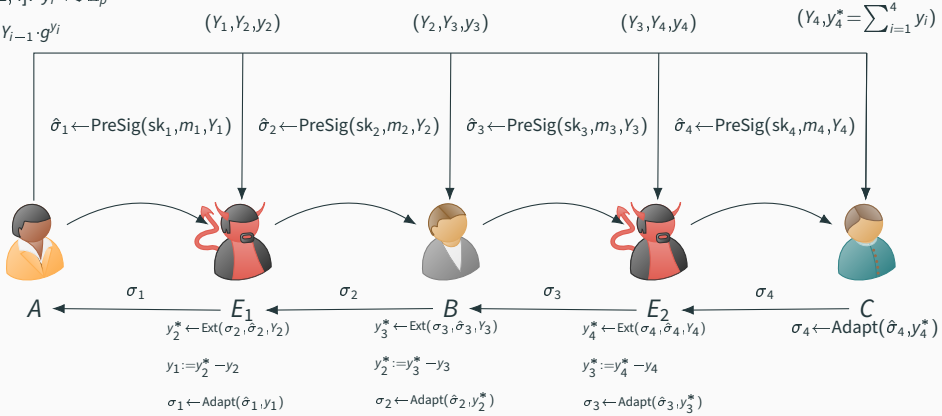
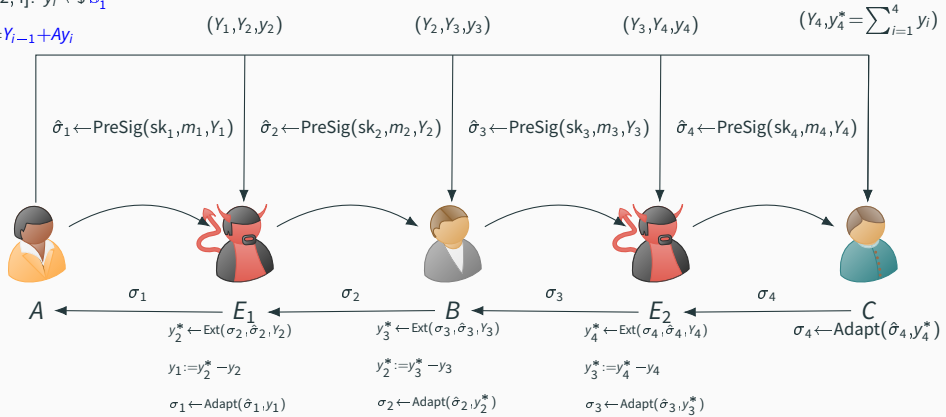


LAS-based PCN

$y_1 \leftarrow\$ \mathbb{S}_1^{n+\ell}$; $Y_1 := Ay_1$

for $i \in [2,4]$: $y_i \leftarrow\$ \mathbb{S}_1^{n+\ell}$

$Y_i := Y_{i-1} + Ay_i$

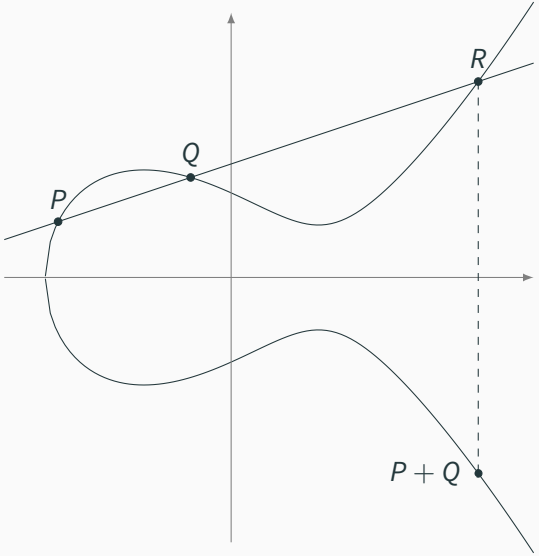


Observation:
The norm of the objects increases as we go along the path.

Background on Isogenies

Elliptic curves[1]

Define a group law such that any three colinear points add up to zero.



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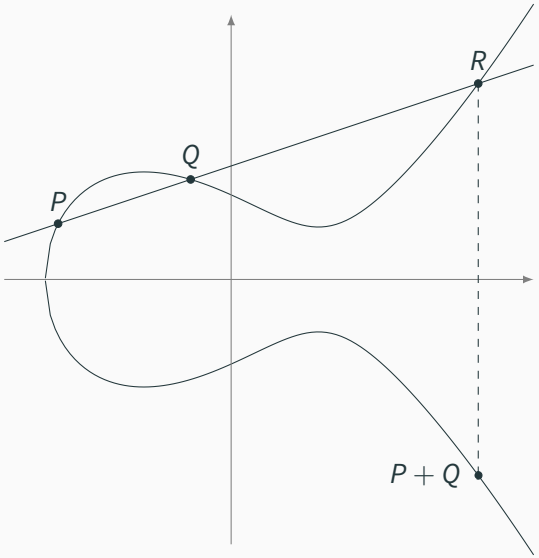
Define a group law such that any three colinear points add up to zero.

- The law is algebraic (it has *formulas*);



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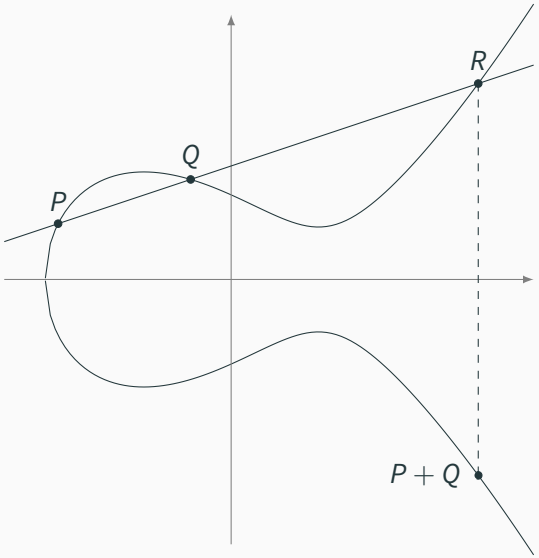
Elliptic curves[1]

Define a group law such that any three colinear points add up to zero.

- The law is algebraic (it has *formulas*);
- The law is commutative;
- $\mathcal{O}$ is the group identity;
- Opposite points have the same x-value.



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Theorem
Let $\phi : E \to E'$ be a map between elliptic curves. These conditions are equivalent:

- ϕ is a surjective group morphism,
- ϕ is a group morphism with finite kernel,
- ϕ is a non-constant algebraic map of projective varieties sending the point at infinity of E onto the point at infinity of E'.

If they hold ϕ is called an isogeny.

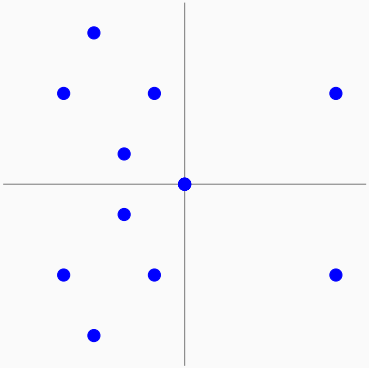
Definition
Let $E : y^2 = x^3 + ax + b$, its ***j*-invariant** is

$$j(E) = 1728\frac{4a^3}{4a^3 + 27b^2}.$$

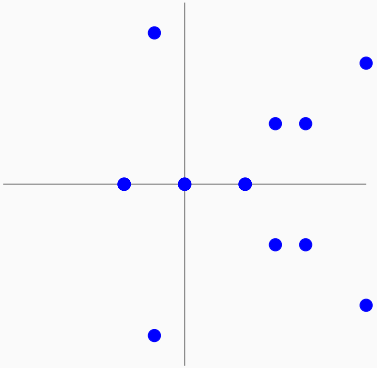
Two elliptic curves E, E' are **isomorphic** if and only if $j(E) = j(E')$.

Isogenies: an example over $\mathbb{F}_{11}$

$E : y^2 = x^3 + x$



$E' : y^2 = x^3 - 4x$



$$\phi(x, y) = \left(\frac{x^2 + 1}{x}, \quad y \frac{x^2 - 1}{x^2} \right)$$

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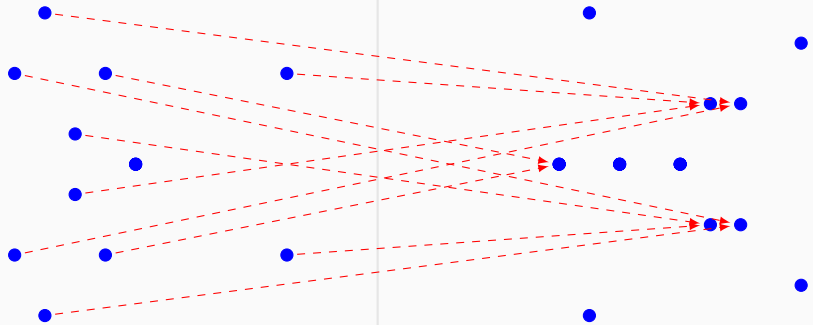

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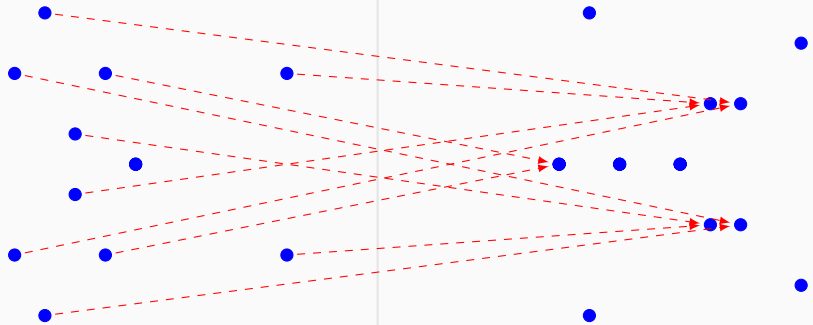

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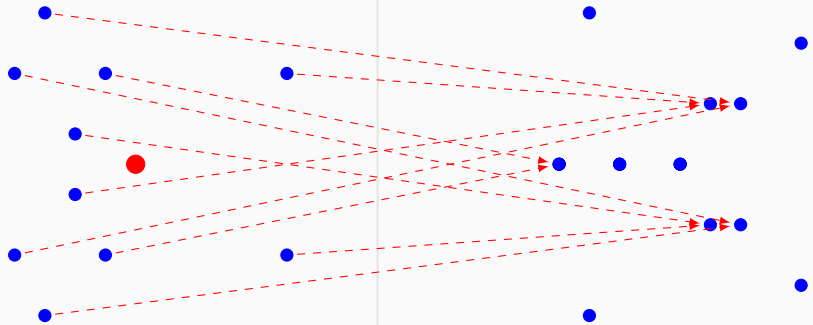

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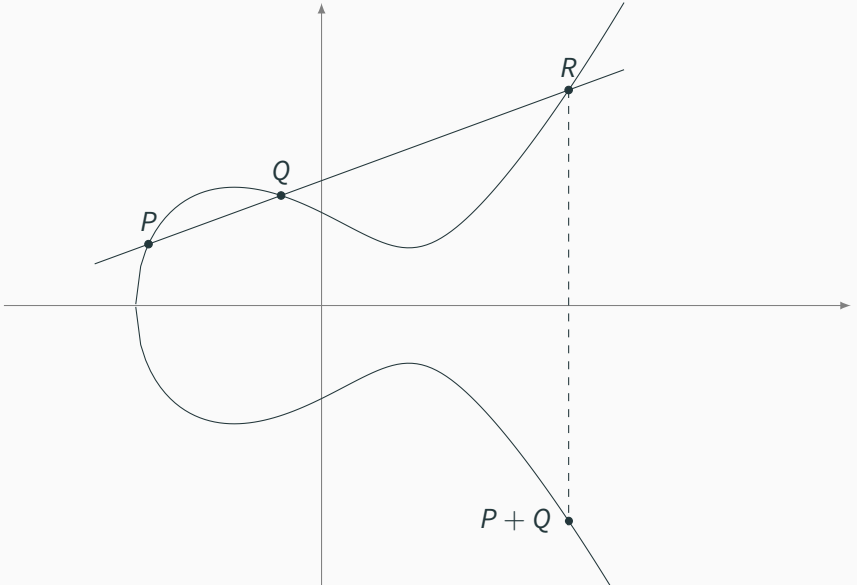

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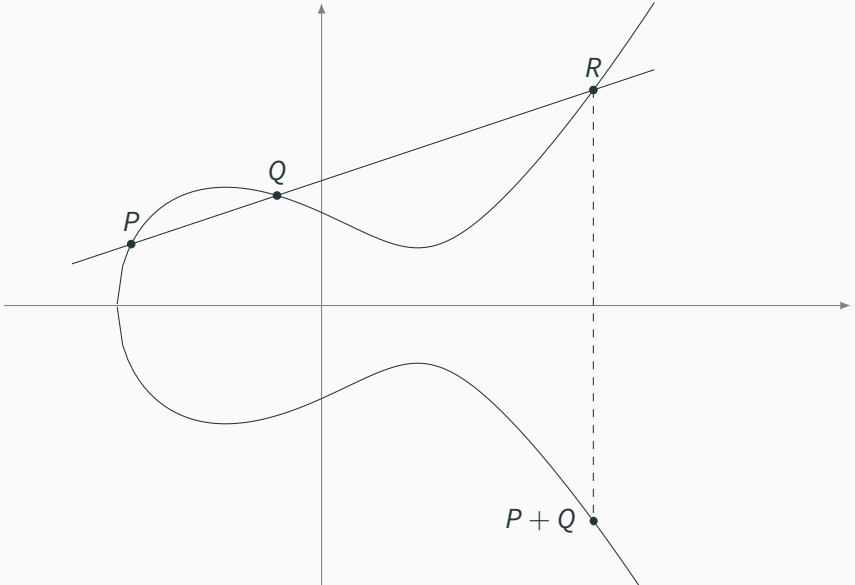

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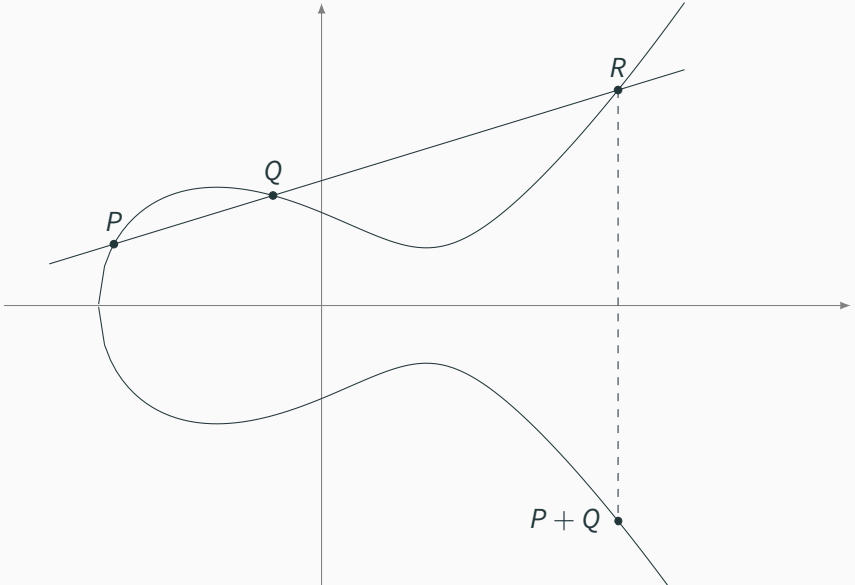

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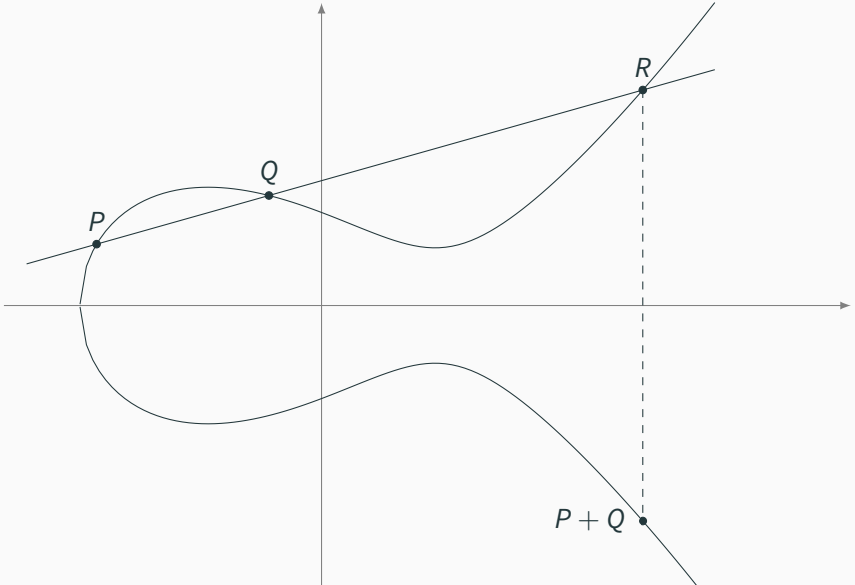

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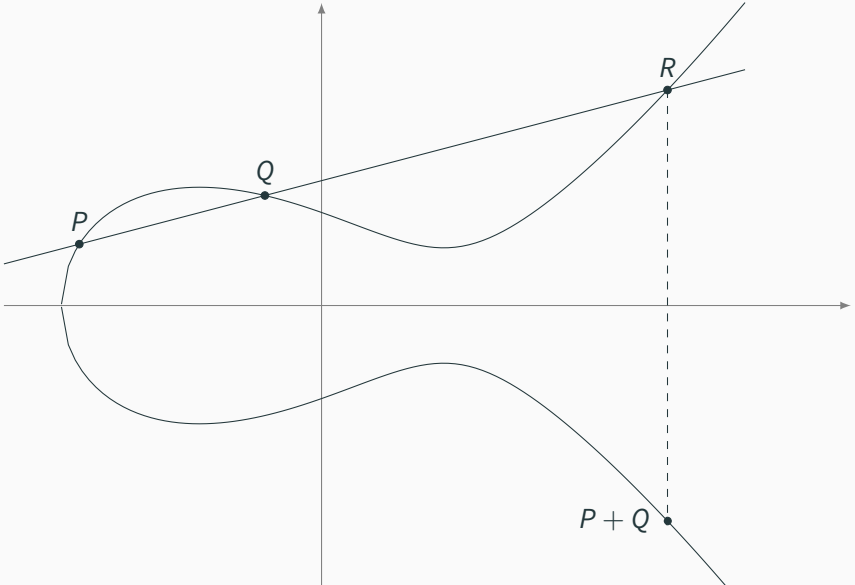

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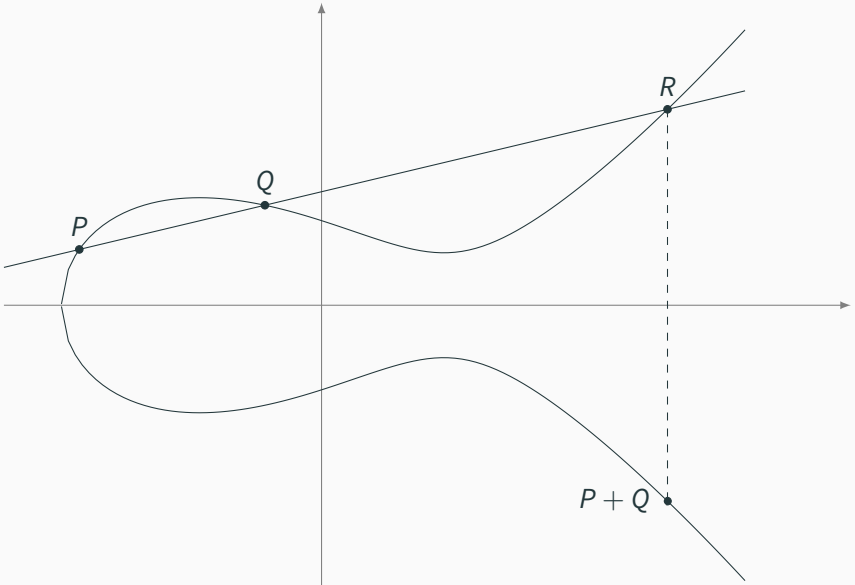

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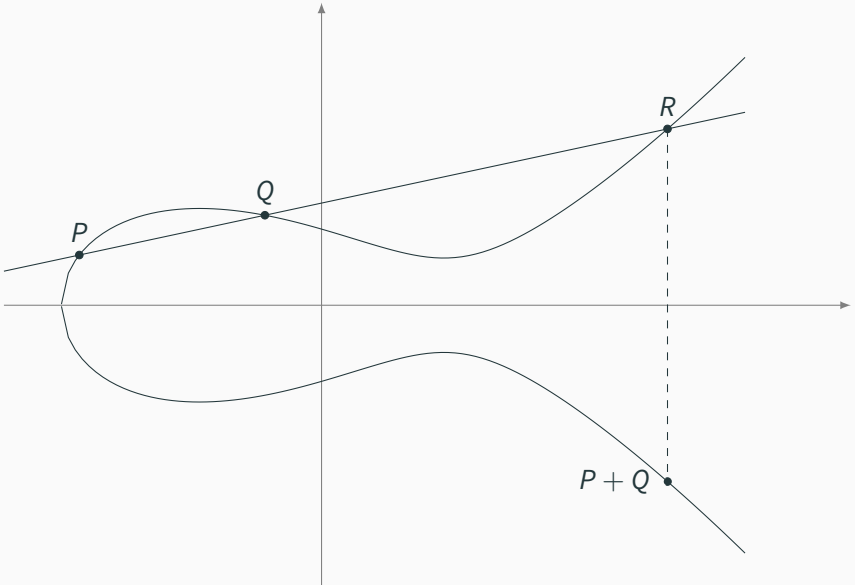

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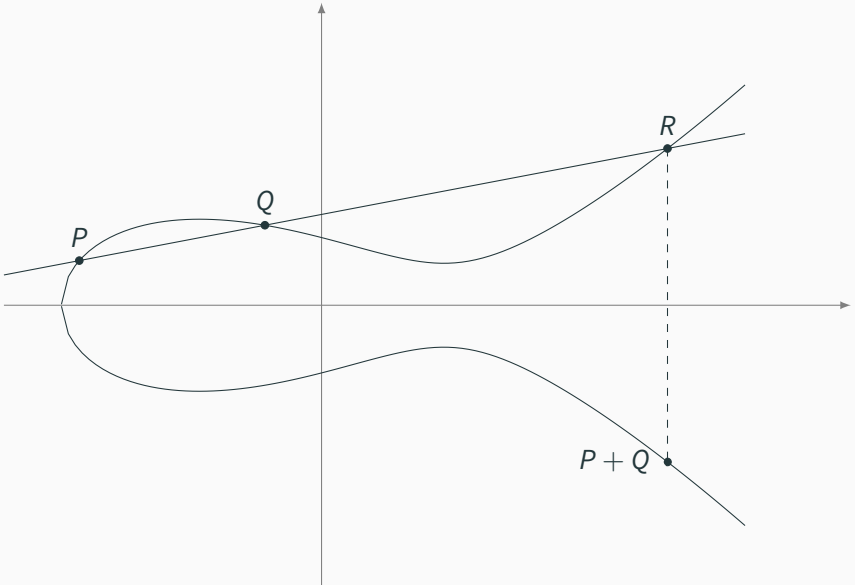

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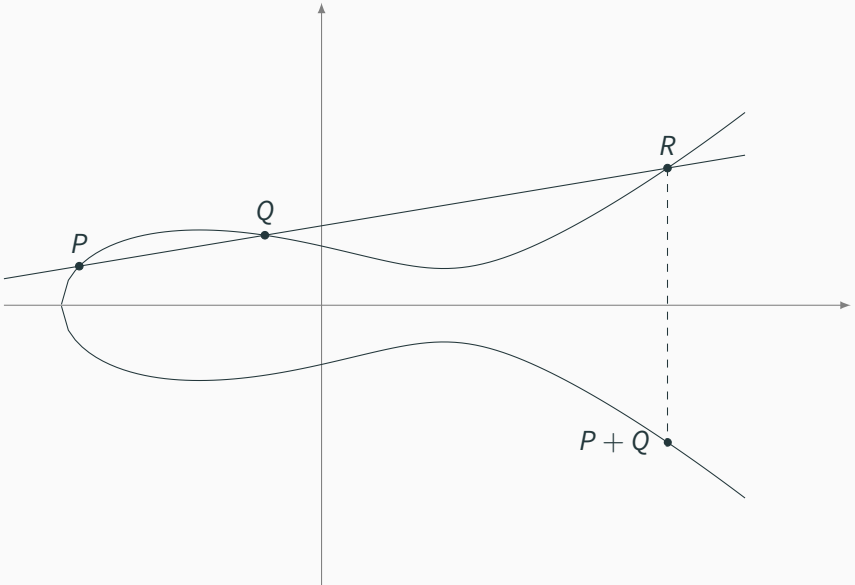

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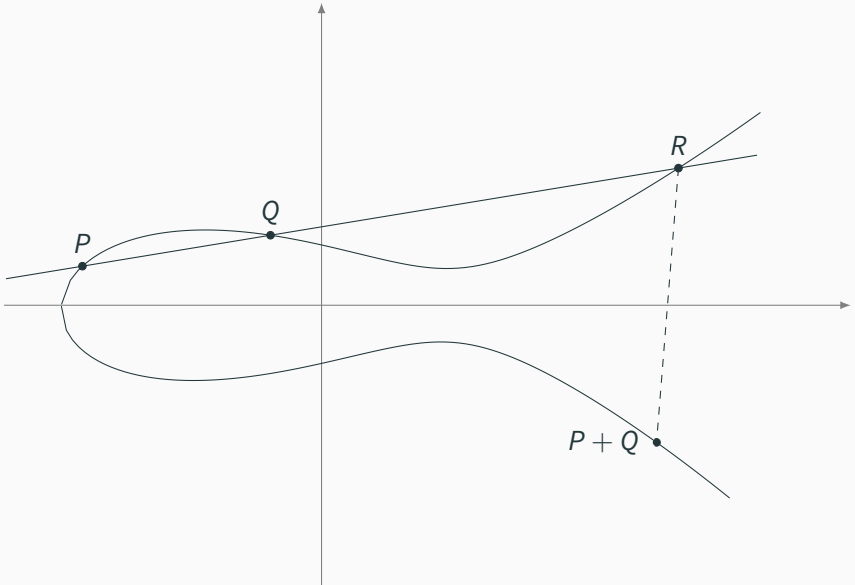

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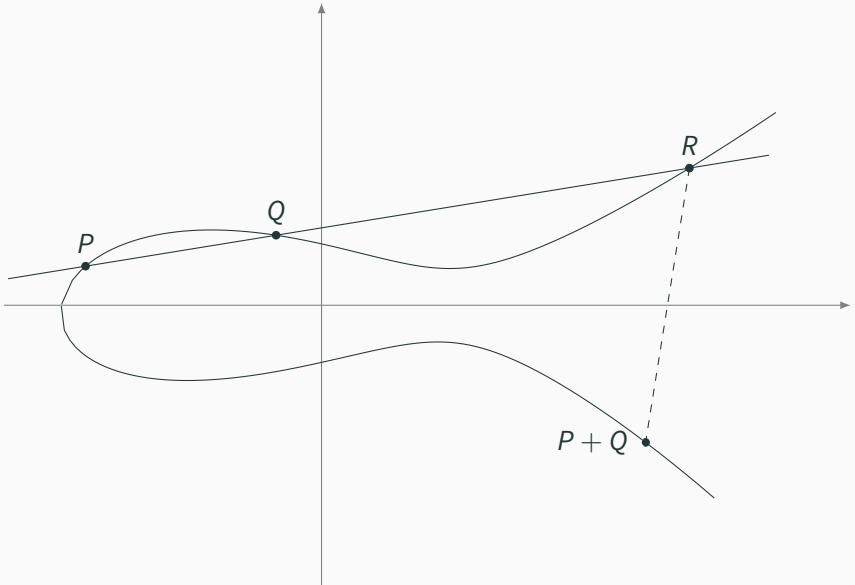

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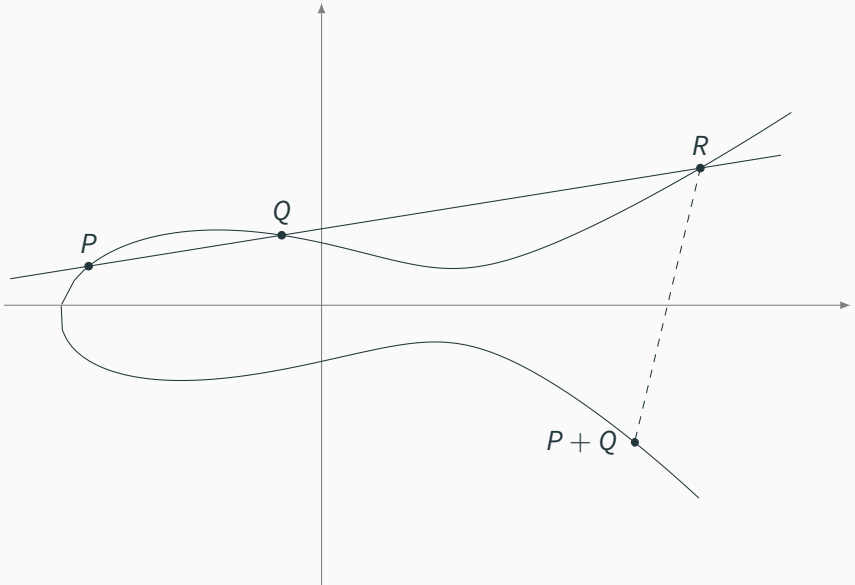

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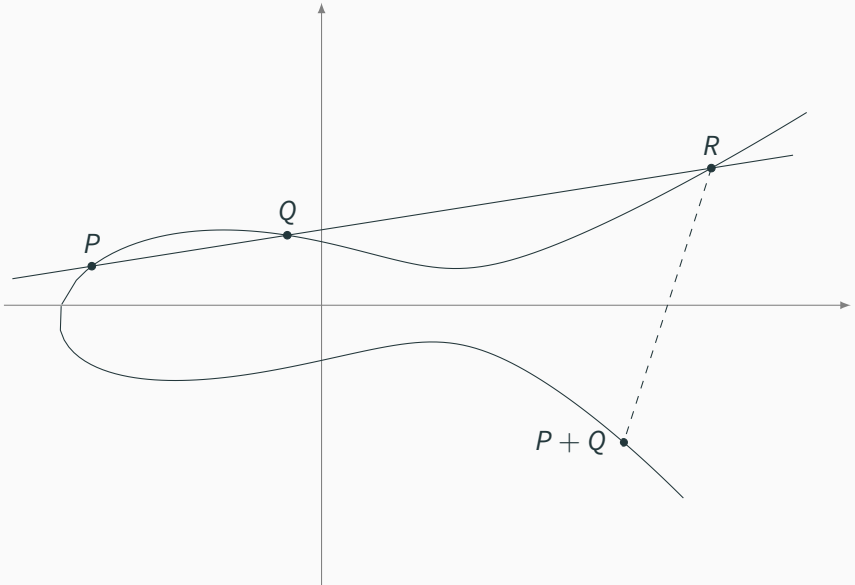

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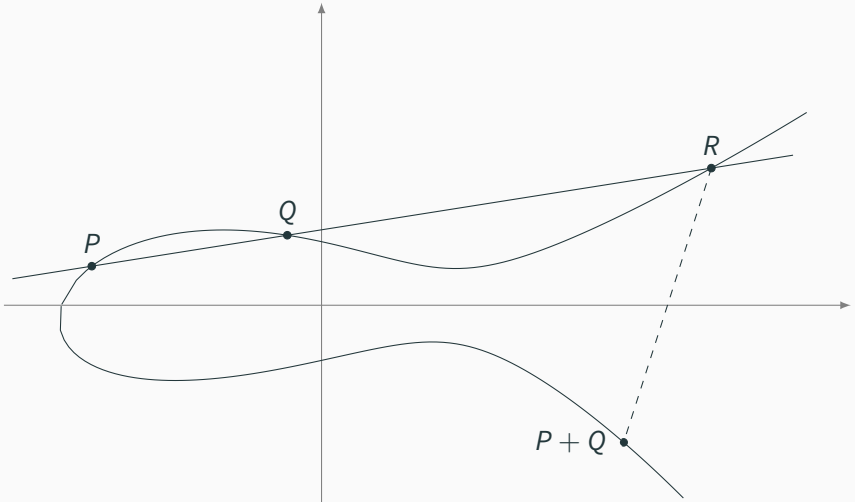

$$y^2 = x^3 + ax + b \quad \longrightarrow \quad j \equiv 1728 \frac{4a^3}{4a^3 + 27b^2}$$

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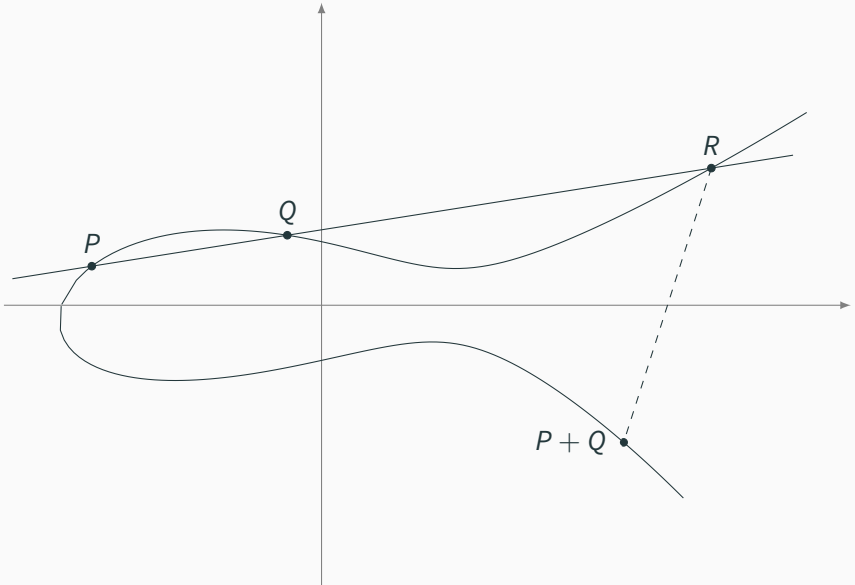

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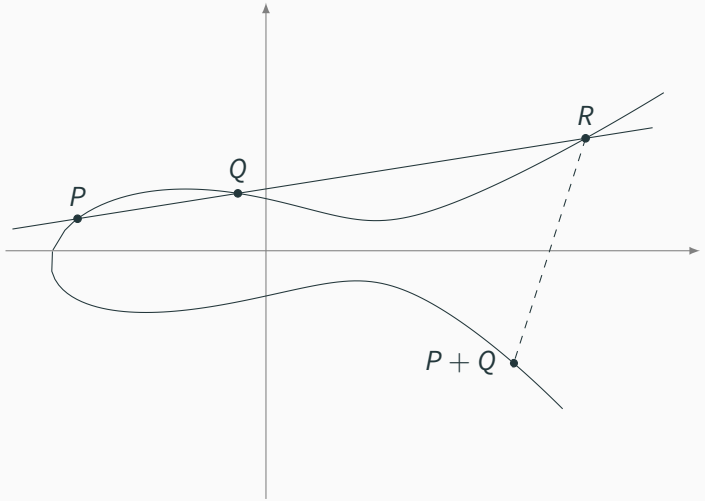

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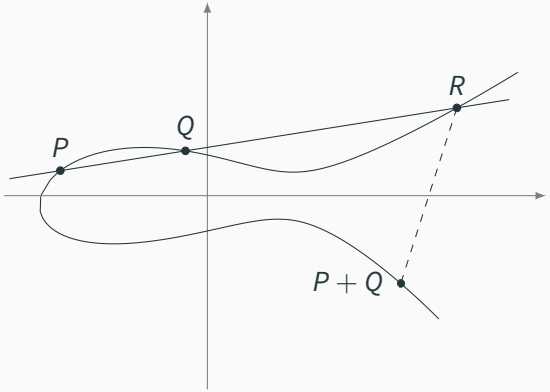

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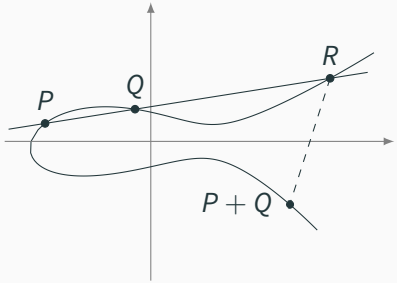

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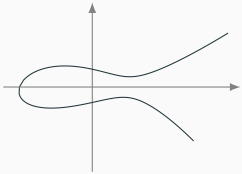

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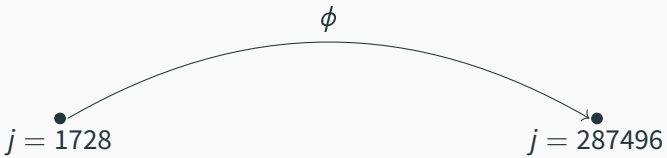
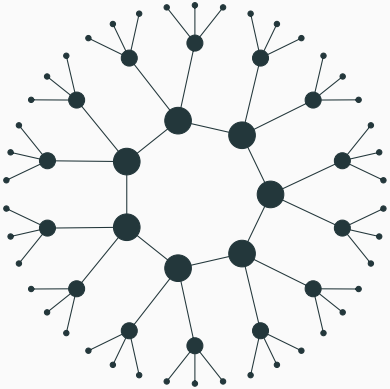


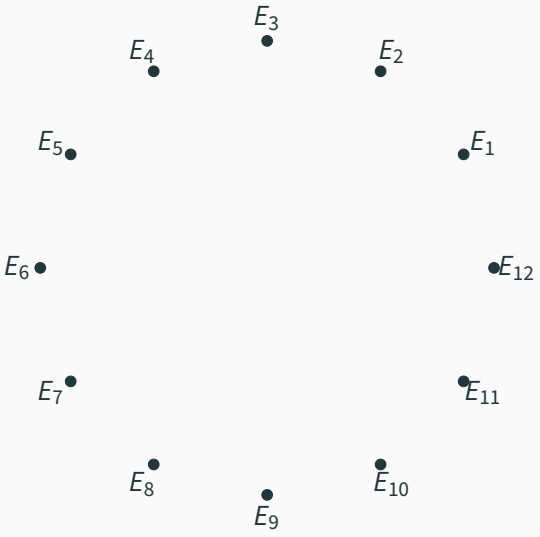
Isogeny graphs

Isogeny graphs

- Vertices are curves up to isomorphism,
- Edges are isogenies up to isomorphism.



Isogeny graph



Vertices are elliptic curves **with complex multiplication by** $\mathcal{O}_K$ (i.e., $\mathrm{End}(E) \simeq \mathcal{O}_K \subset \mathbb{Q}(\sqrt{-D})$).

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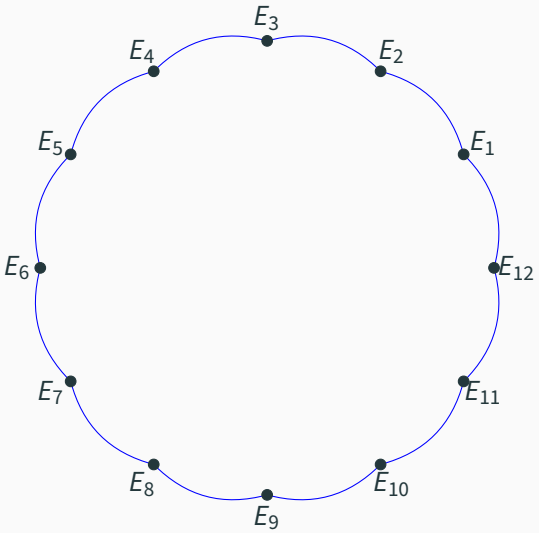

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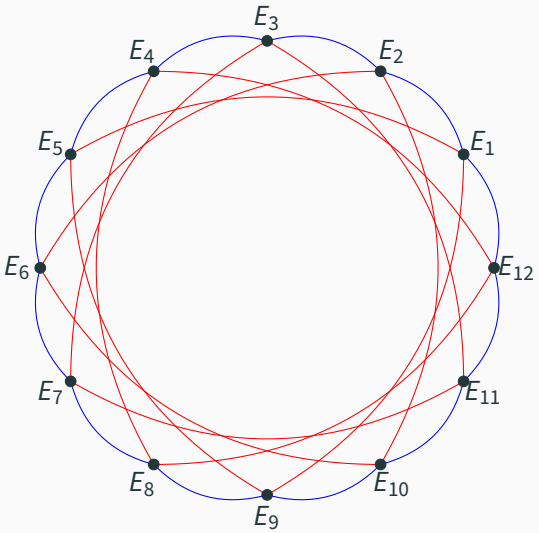

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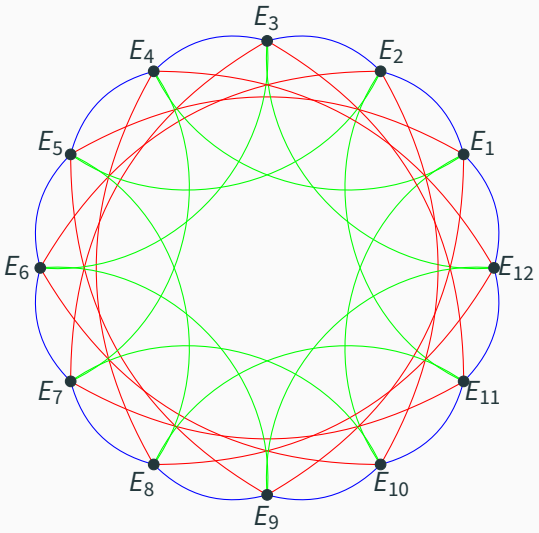

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Class group action

Definition
The **class group** of an order $\mathcal{O} \subset \mathbb{Q}(\sqrt{-D})$ is the quotient $\mathrm{Cl}(\mathcal{O}) = \mathcal{I}(\mathcal{O})/\mathcal{P}(\mathcal{O})$. It is a **finite abelian** group.

Theorem (Main theorem of complex multiplication)
The class group of $\mathcal{O}$ acts **faithfully and transitively** on the set of elliptic curves with CM by $\mathcal{O}$ (denoted by $\mathrm{Ell}(\mathcal{O})$) with the **group action** as:

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Observation:
There is no meaningful operation between two elements of $\mathrm{Ell}(\mathcal{O})$.

Definition (Group Action Inversion Problem (GAIP) [DFG19])
Given two elliptic curves E and E' over the same finite field and with $\mathrm{End}(E) = \mathrm{End}(E') = \mathcal{O}$, find an ideal $\mathfrak{a} \subset \mathcal{O}$ such that $E' = \mathfrak{a} \star E$.

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Definition (Isogeny walk problem)
Given two isogenous elliptic curves E and E' over the same finite field, find a path $E \to E'$ in an isogeny graph.

Isogeny-based Adaptor Signature

Identification scheme from isogenies

E_0 is a designated base (starting) curve that is part of public parameters.

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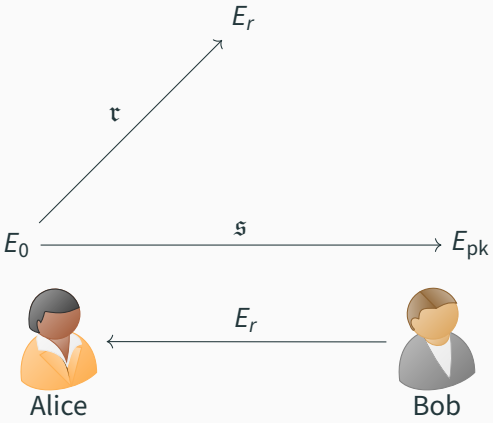

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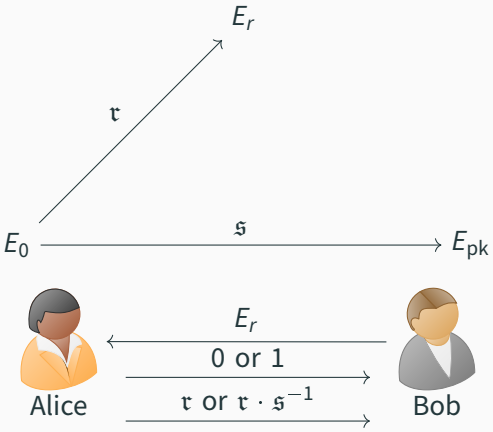

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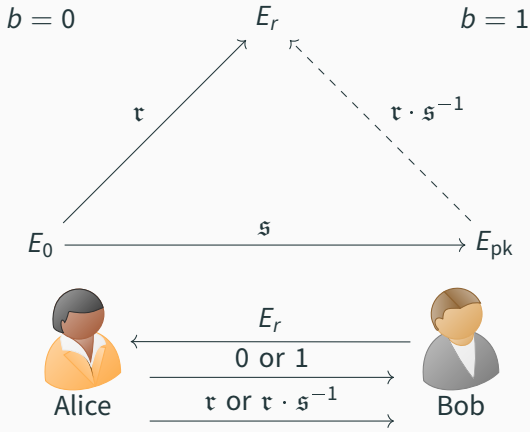

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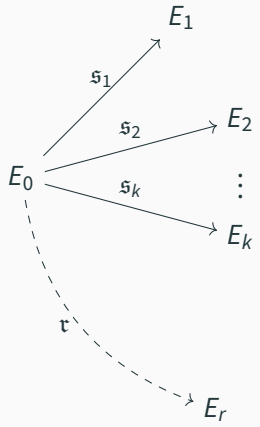

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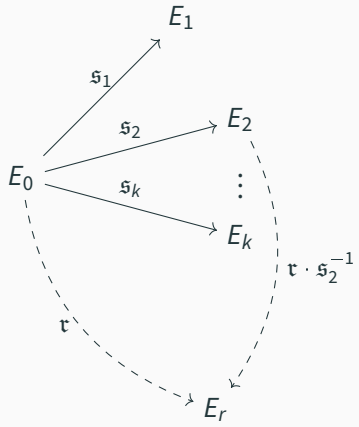

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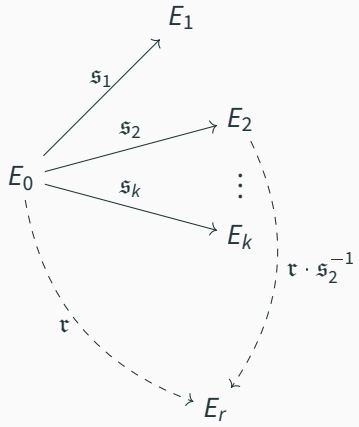

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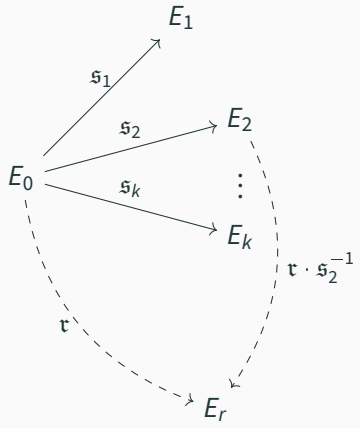
- To achieve security level λ, we require $t = \frac{\lambda}{\log_2 S}$ iterations.



Isogeny Adaptor Signature (IAS)

- Combining the previous approaches with Fiat-Shamir transform we obtain the isogeny-based signature scheme CSI-FiSh [BKV19]
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Schnorr

procedure PreSig(sk, m, Y)

$k \leftarrow_\$ \mathbb{Z}_p, R := g^k$

$e := H(\mathsf{pk} \| R \cdot Y \| m)$

$\hat{s} := k + e \cdot \mathsf{sk}$

return $\hat{\sigma} := (e, \hat{s})$

IAS [TMSM20]

procedure PreSig(sk, m, E_Y)

$\mathfrak{k} \leftarrow_\$ \mathrm{Cl}(\mathcal{O}), E_R := \mathfrak{k} \star E_0$

$e := H(\mathsf{pk} \| E_R \cdot E_Y \| m)$

$\hat{\mathfrak{s}} := \mathfrak{k} - e \cdot \mathsf{sk}$

return $\hat{\sigma} := (e, \hat{\mathfrak{s}})$

Isogeny Adaptor Signature (IAS)

- Combining the previous approaches with Fiat-Shamir transform we obtain the isogeny-based signature scheme CSI-FiSh [BKV19]
- We can construct an adaptor signature from CSI-FiSh using GAIP as the hard relation (for simplicity we consider the base scheme with challenge space $\{0, 1\}$).

Schnorr

procedure PreSig(sk, m, Y)

$k \leftarrow_\$ \mathbb{Z}_p, R := g^k$

$e := H(\mathsf{pk} \| R \cdot Y \| m)$

$\hat{s} := k + e \cdot \mathsf{sk}$

return $\hat{\sigma} := (e, \hat{s})$

IAS [TMSM20]

procedure PreSig(sk, m, E_Y)

$\mathfrak{k} \leftarrow_\$ \mathrm{Cl}(\mathcal{O}), E_R := \mathfrak{k} \star E_0$

$e := H(\mathsf{pk} \| E_R \cdot E_Y \| m)$

$\hat{\mathfrak{s}} := \mathfrak{k} - e \cdot \mathsf{sk}$

return $\hat{\sigma} := (e, \hat{\mathfrak{s}})$

Observation:
We cannot combine E_R and E_Y as there is no meaningful operation between two elliptic curves from $\mathrm{Ell}(\mathcal{O})$

Solution:
Randomize the statement E_Y with the group action of E_R and prove the relation between E_R and E_Y in ZK (i.e., a DH-tuple proof for isogenies) [TMSM20].

Solution:
Randomize the statement E_Y with the group action of E_R and prove the relation between E_R and E_Y in ZK (i.e., a DH-tuple proof for isogenies) [TMSM20].

procedure PreSig(sk, m, E_Y)

$\mathfrak{k} \leftarrow\!\!\$\ \mathsf{Cl}(\mathcal{O})$; $E_R := \mathfrak{k} \star E_0$; $\hat{E}_R := \mathfrak{k} \star E_Y$

Set $x := (E_0, E_R, E_Y, \hat{E}_R)$

$\pi \leftarrow \mathsf{P}_{\mathsf{NIZK}}(x, \mathfrak{k})$

$e := H(\mathsf{pk} \| \hat{E}_R \| m)$

$\hat{\mathfrak{s}} := \mathfrak{k} - e \cdot \mathsf{sk}$

return $\hat{\sigma} := (e, \hat{\mathfrak{s}}, \hat{E}_R, \pi)$

Solution:
Randomize the statement E_Y with the group action of E_R and prove the relation between E_R and E_Y in ZK (i.e., a DH-tuple proof for isogenies) [TMSM20].

procedure PreSig(sk, m, E_Y)

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$\pi \leftarrow \mathsf{P}_{\mathsf{NIZK}}(x, \mathfrak{k})$

$e := H(\mathsf{pk} \| \hat{E}_R \| m)$

$\hat{\mathfrak{s}} := \mathfrak{k} - e \cdot \mathsf{sk}$

return $\hat{\sigma} := (e, \hat{\mathfrak{s}}, \hat{E}_R, \pi)$

procedure PreVer(pk, m, E_Y, $\hat{\sigma}$)

Parse pk as (E_0, E_1)

Parse $\hat{\sigma}$ as $(e, \hat{\mathfrak{s}}, \hat{E}_R, \pi)$

$E_R := \hat{\mathfrak{s}} \star \mathsf{pk}_e$

Set $x := (E_0, E_R, E_Y, \hat{E}_R)$

if $\pi \leftarrow \mathsf{V}_{\mathsf{NIZK}}(x, \pi) \neq 1$ **then**

return 0

$e' = H(\mathsf{pk} \| \hat{E}_R \| m)$

return $(e = e')$

Solution:
Randomize the statement E_Y with the group action of E_R and prove the relation between E_R and E_Y in ZK (i.e., a DH-tuple proof for isogenies) [TMSM20].

procedure PreSig(sk, m, E_Y)
- $\mathfrak{k} \leftarrow_{\$} \mathrm{Cl}(\mathcal{O})$; $E_R := \mathfrak{k} \star E_0$; $\hat{E}_R := \mathfrak{k} \star E_Y$
- Set $x := (E_0, E_R, E_Y, \hat{E}_R)$
- $\pi \leftarrow \mathsf{P}_{\mathsf{NIZK}}(x, \mathfrak{k})$
- $e := H(\mathsf{pk} \| \hat{E}_R \| m)$
- $\hat{\mathfrak{s}} := \mathfrak{k} - e \cdot \mathsf{sk}$
- **return** $\hat{\sigma} := (e, \hat{\mathfrak{s}}, \hat{E}_R, \pi)$

procedure PreVer(pk, m, E_Y, $\hat{\sigma}$)
- Parse pk as (E_0, E_1)
- Parse $\hat{\sigma}$ as $(e, \hat{\mathfrak{s}}, \hat{E}_R, \pi)$
- $E_R := \hat{\mathfrak{s}} \star \mathsf{pk}_e$
- Set $x := (E_0, E_R, E_Y, \hat{E}_R)$
- **if** $\pi \leftarrow \mathsf{V}_{\mathsf{NIZK}}(x, \pi) \neq 1$ **then**
  - **return** 0
- $e' = H(\mathsf{pk} \| \hat{E}_R \| m)$
- **return** $(e = e')$

Adapt and Ext algorithms are analogous to Schnorr-based construction.

Zero-knowledge proof for group action

Cozzo and Smart [CS20] showed how to prove knowledge of a secret isogeny generically. More precisely, they showed a zero-knowledge proof for the following relation (i.e., knowledge of a witness $\mathfrak{s}$ for j simultaneous instances of the GAIP):

$$L_j := \left\{ \left((E_1, E_1', \ldots, E_j, E_j'), \mathfrak{s} \right) : \bigwedge_{i=1}^{j} (E_i' = \mathfrak{s} \star E_i) \right\}.$$

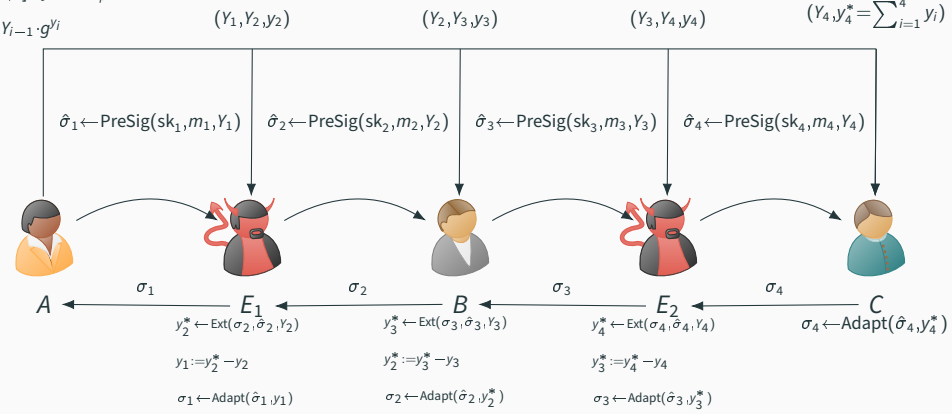
The proof simply involves running multiple iterations of the basic identification scheme to amplify the soundness. We note that we only need this for L_2.

IAS-based PCN

$y_1 \leftarrow\$ \mathbb{Z}_p$; $Y_1 := g^{y_1}$

for $i \in [2,4]$: $y_i \leftarrow\$ \mathbb{Z}_p$

$Y_i := Y_{i-1} \cdot g^{y_i}$

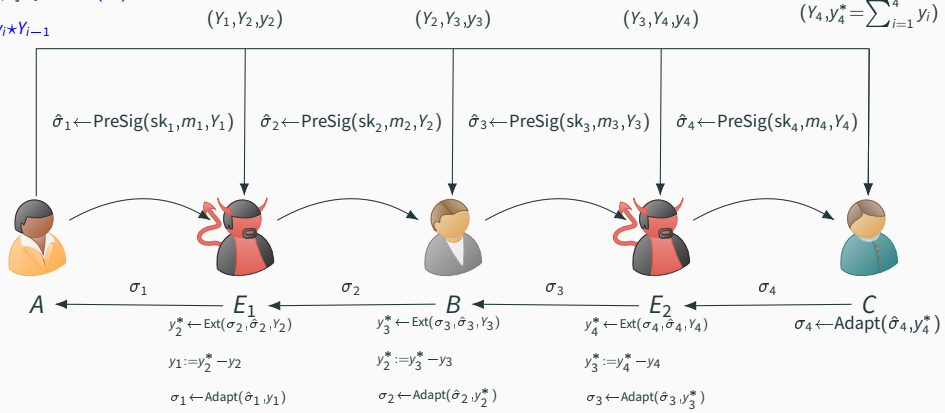


IAS-based PCN

$y_1 \leftarrow\$ \mathrm{Cl}(\mathcal{O})$; $Y_1 := y_1 \star E_0$

for $i \in [2,4]$: $y_i \leftarrow\$ \mathrm{Cl}(\mathcal{O})$

$Y_i := y_i \star Y_{i-1}$



Performance Evaluation

- C implementation, parallelized with OpenMP. Benchmarked on a KVM-based VM with 2.0GHz AMD EPYC 7702 processor with 16 cores and 32GB RAM, running Ubuntu 18.04 LTS
- Source code: `https://github.com/etairi/Adaptor-CSI-FiSh`

S	t_S	k	$\lvert sk \rvert$	$\lvert pk \rvert$	$\lvert \hat{\sigma} \rvert$	$\lvert \sigma \rvert$	KGen	Sig	Ver	PreSig	PreVer	Ext	Adapt
2^1	56	16	16	128	19944	1880	0.05	0.24	0.23	3.59	3.55	0.005	0.005
2^2	38	14	16	256	19672	1286	0.06	0.16	0.16	2.75	2.68	0.005	0.005
2^3	28	16	16	512	19020	956	0.07	0.13	0.14	2.21	2.15	0.005	0.005
2^4	23	13	16	1024	19338	791	0.07	0.11	0.11	1.99	1.94	0.005	0.005
2^6	16	16	16	4096	18624	560	0.29	0.08	0.09	1.61	1.56	0.005	0.005
2^8	13	11	16	16384	19330	461	1.00	0.08	0.08	1.50	1.44	0.005	0.005
2^{10}	11	7	16	65536	19908	395	3.21	0.06	0.06	1.40	1.36	0.005	0.005
2^{12}	9	11	16	262144	19198	329	12.89	0.06	0.06	1.30	1.25	0.005	0.005
2^{15}	7	16	16	2097152	18327	263	102.02	0.06	0.06	1.16	1.11	0.005	0.005

Open Problems

- Can we construct adaptor signatures from other post-quantum assumptions (e.g., multivariate quadratics, code-based, hash-based)?
- Can we transform SQISign [DFKL$^+$20] into an adaptor signature to obtain a more efficient isogeny-based construction?


Thank you!
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