Everything is a Race and Nakamoto Always Wins

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Main Results

- Nakamoto's private double-spend attack is the worst attack: Bitcoin is secure if the private attack fails with high probability.

- Maximum tolerable adversary power is determined by the private attack threshold.

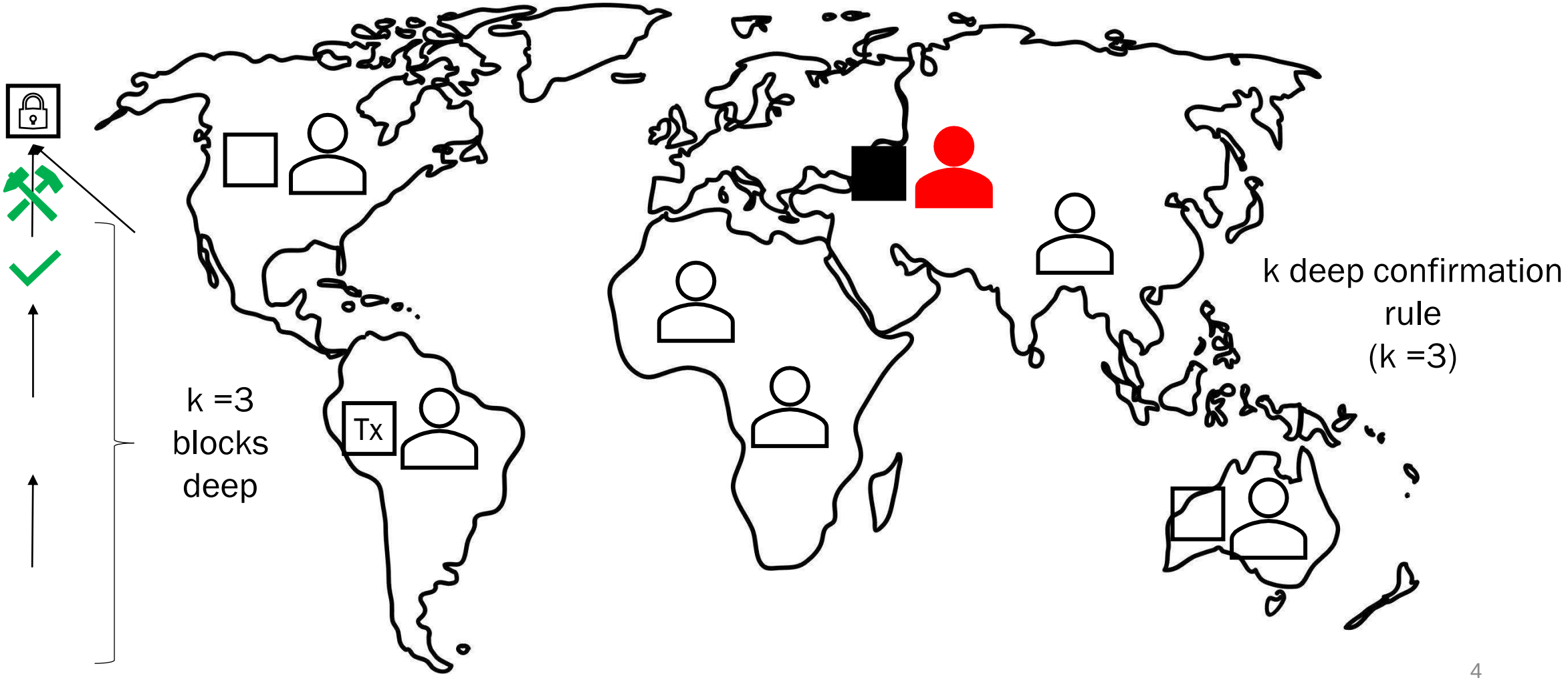
- This is true for three classes of longest chain protocols:
  - ✓ Nakamoto's original Proof-of-Work protocol
  - ✓ Ouroboros and Sleepy Proof-of-Stake protocols
  - ✓ Chia Proof-of-Space protocol

Let’s start with Nakamoto

Bitcoin: A Peer-to-Peer Electronic Cash System

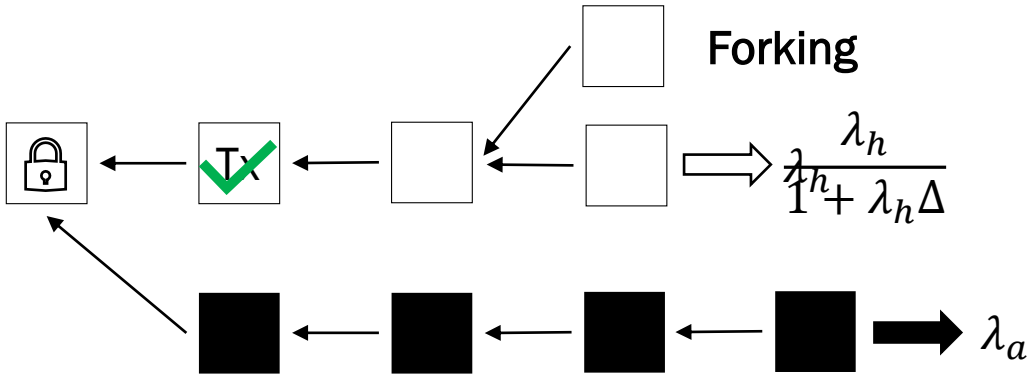
Satoshi Nakamoto
satoshin@gmx.com
www.bitcoin.org

Let's start with Nakamoto



Nakamoto's Private Attack

k deep confirmation rule
(k =3)



Private attack succeeds!

Safety: Once Tx is k-deep, Tx stays as part of the longest chain.

Liveness: Longest chain contains honest blocks.

Security

Private attack fails if $\lambda_a < \frac{\lambda_h}{1+\lambda_h\Delta}$

Private attack succeeds if $\lambda_a > \frac{\lambda_h}{1+\lambda_h\Delta}$

Can another attack succeed?

Network Model

- Synchronous network with delay Δ
- Total mining rate: λ
- Byzantine Adversary with mining rate $\lambda_a = \beta\lambda$
- Honest mining rate: $\lambda_h = \lambda - \lambda_a = (1-\beta)\lambda$
- Blocks arrive via a Poisson process of rate λ (in the limit size of time slots approach zero).

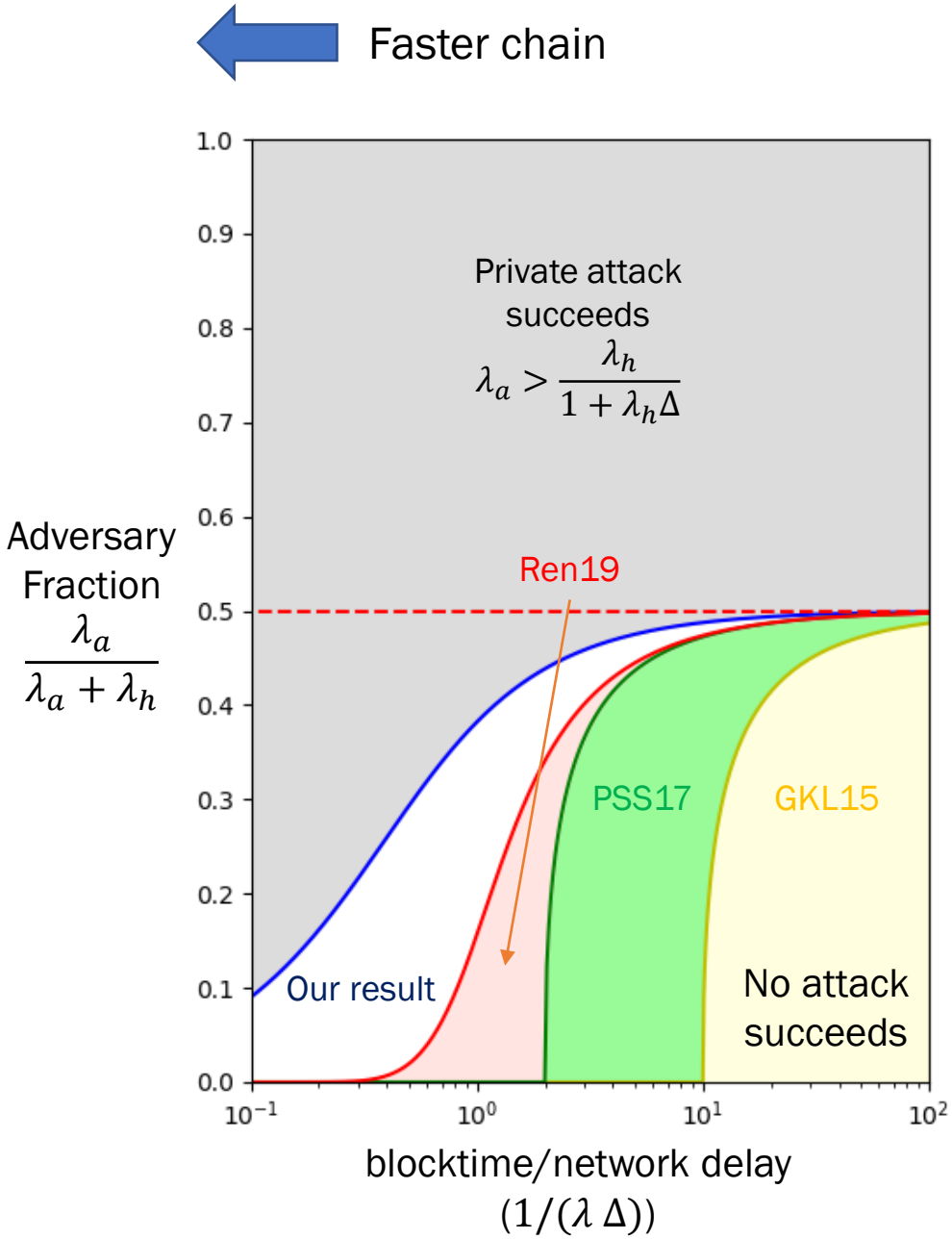
Closing the gap

Given $\beta = \lambda_a/(\lambda_a + \lambda_h)$ and Δ, for what values of blocktime $(1/\lambda)$, is Nakamoto's PoW longest chain protocol secure?

Nakamoto's longest chain protocol is secure if $\lambda_a < \frac{\lambda_h}{1+\lambda_h\Delta}$.

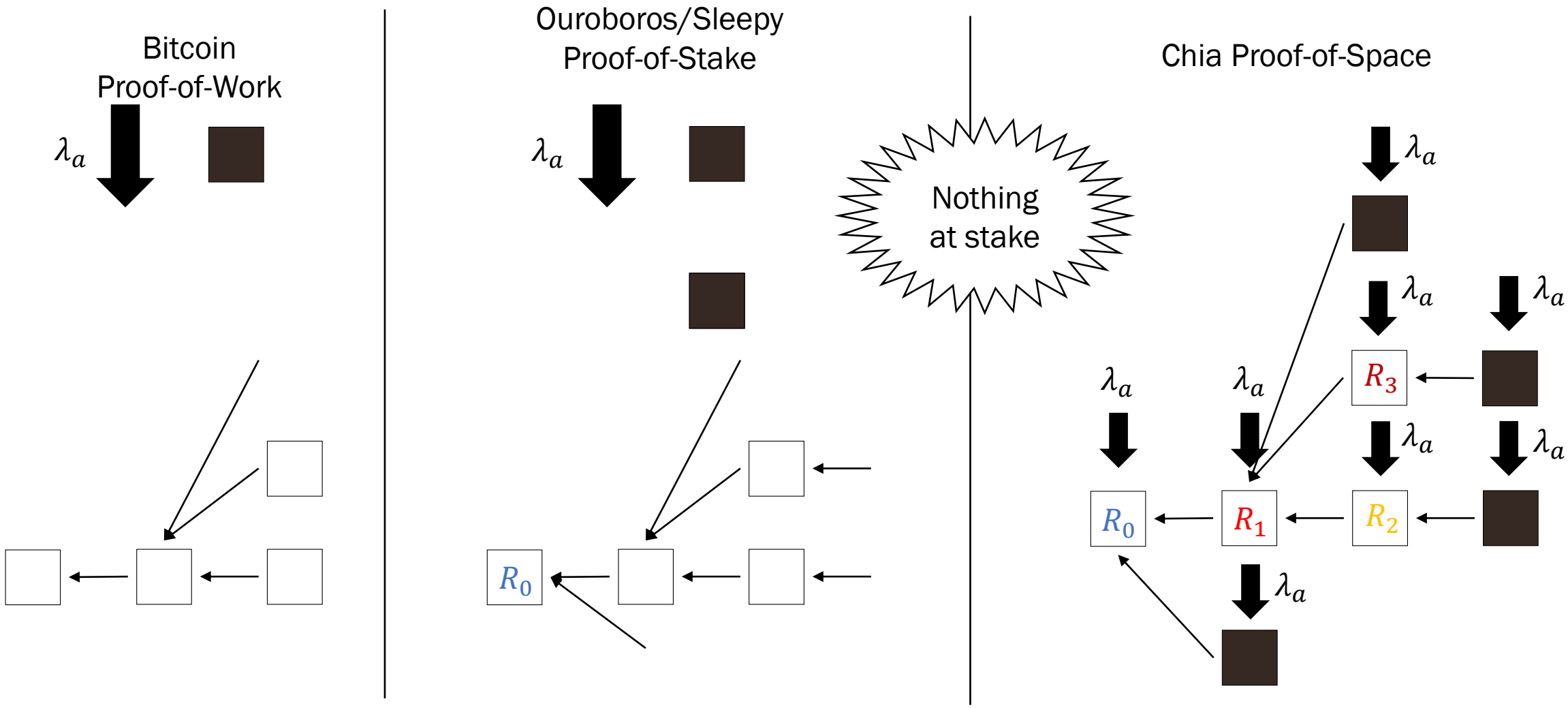
If $\lambda_a > \frac{\lambda_h}{1+\lambda_h\Delta}$, private attack succeeds.

Smaller blocktime = Faster chain!!



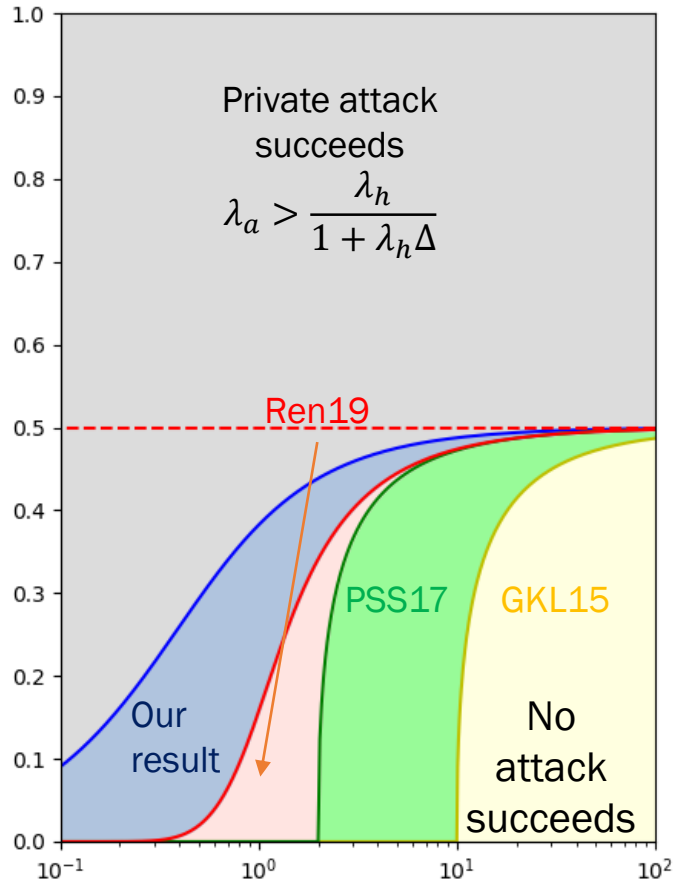
Other longest chain protocols

DGKR'17, DPC'19
CP'19

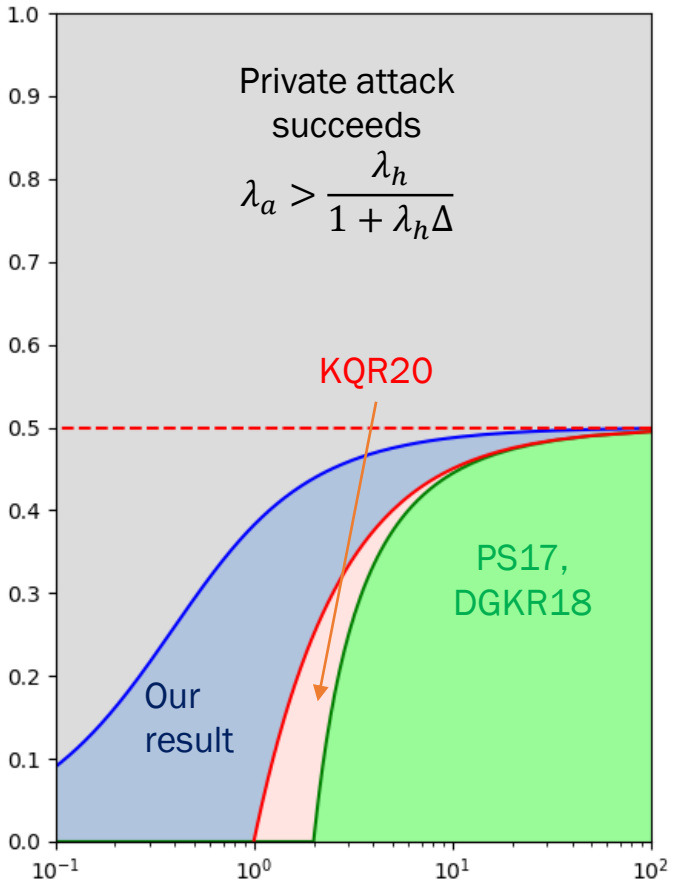


More results

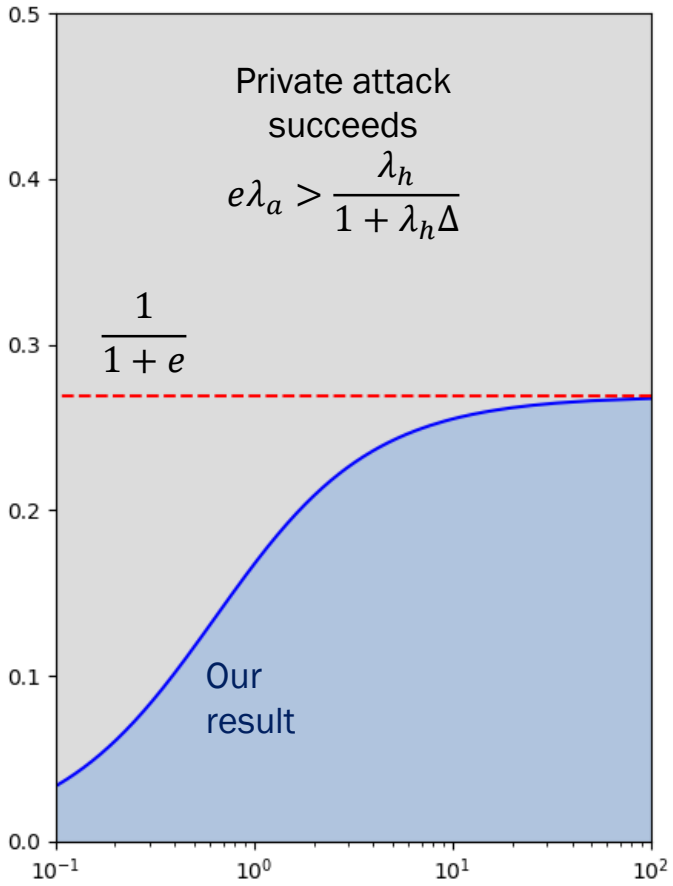
Proof-of-Work



Ouroboros/Sleepy



Chia Proof-of-Space

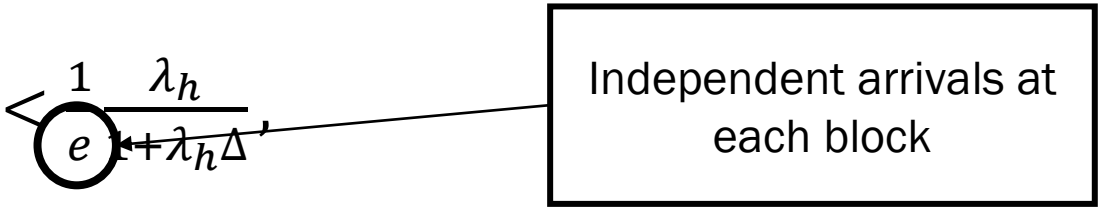


Main Theorems

- If $\lambda_a < \frac{\lambda_h}{1+\lambda_h \Delta}$,

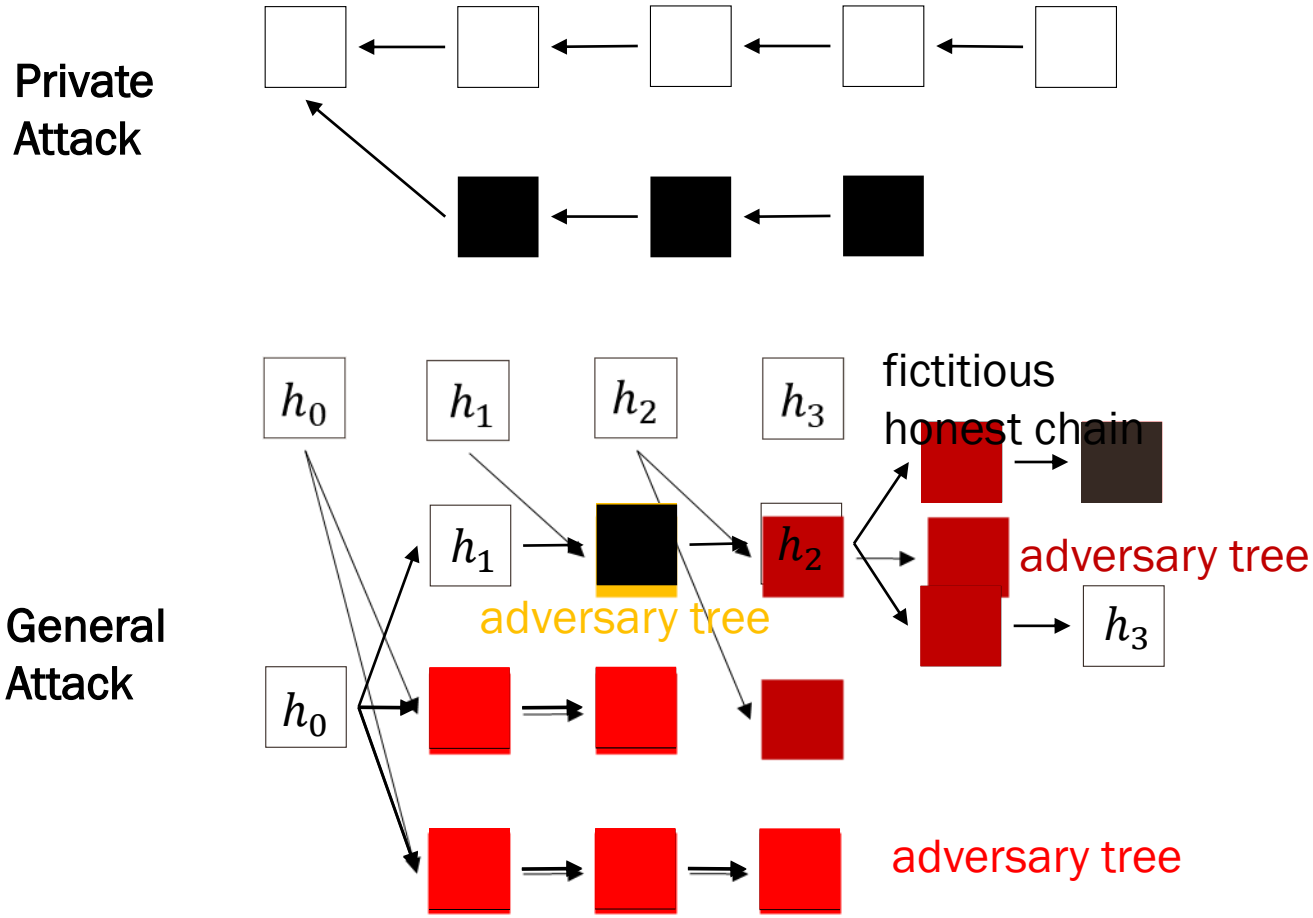
then the **Nakamoto Proof-of-Work** and the **Ouroboros/Sleepy Proof-of-Stake** protocols generate transaction ledgers such that each transaction *tx* satisfies safety and liveness (parameterized by σ) with probability at least $1 - e^{-\Omega(\sigma^{1-\varepsilon})}$, for any $\varepsilon > 0$.

- If $\lambda_a < \frac{1}{e} \frac{\lambda_h}{1+\lambda_h \Delta}$,

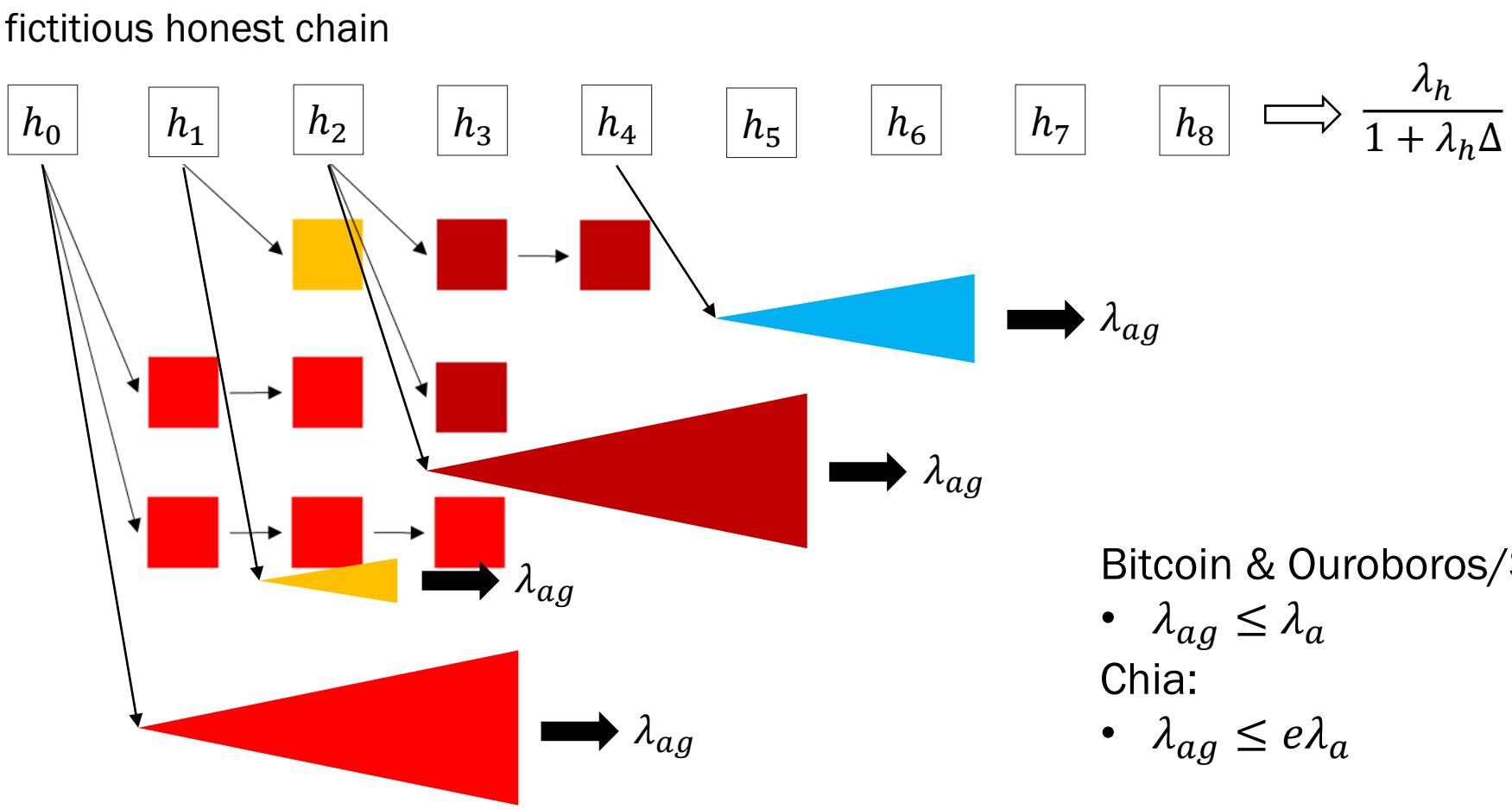


then the **Chia Proof-of-Space** protocol generates transaction ledgers such that each transaction *tx* satisfies safety and liveness (parameterized by σ) with probability at least $1 - e^{-\Omega(\sigma^{1-\varepsilon})}$, for any $\varepsilon > 0$.

Blocktree partitioning



Everything is a race



Bitcoin & Ouroboros/Sleepy:

- $\lambda_{ag} \leq \lambda_a$

Chia:

- $\lambda_{ag} \leq e\lambda_a$

And Nakamoto Always Wins

Nakamoto blocks stabilize its prefix!

Secure against private attack

$$\Downarrow$$

$$\frac{\lambda_h}{1+\lambda_h\Delta} > \lambda_{ag}$$

$$\Downarrow$$

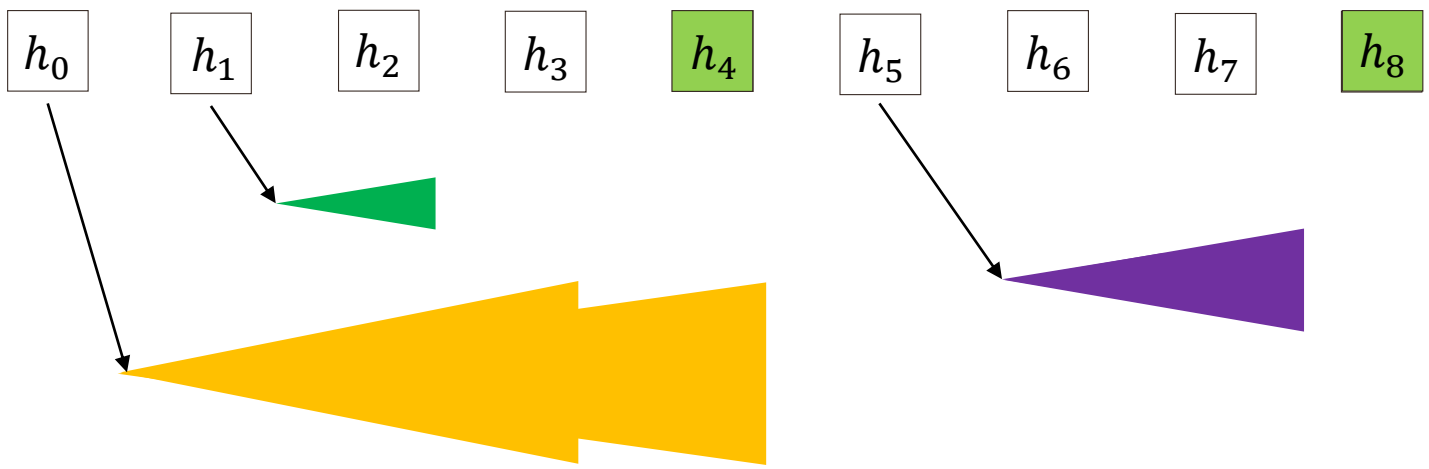
Nakamoto blocks exist and keep arriving.

$$\Downarrow$$

secure against all attacks

$$\Rightarrow \frac{\lambda_h}{1+\lambda_h\Delta}$$

fictitious honest chain



$$\Rightarrow \lambda_{ag}$$

Bitcoin & Ouroboros/SnowWhite:

- $\lambda_{ag} \leq \lambda_a$

Chia:

- $\lambda_{ag} \leq e\lambda_a$

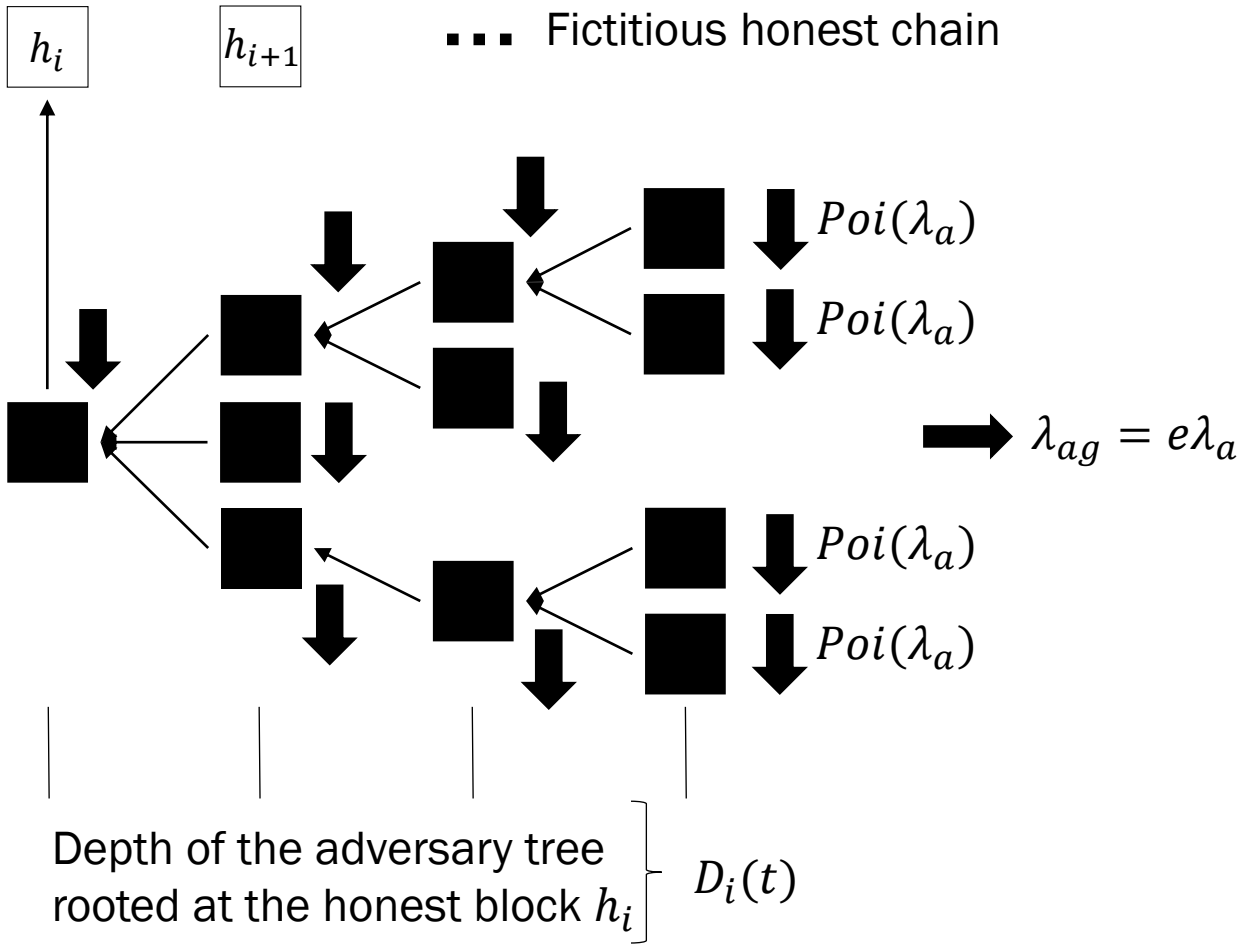
Other Methods

Method	Protocol	Work	Property
Backbone Analysis (block counting)	Nakamoto Proof-of-Work	GKL'15	First to show security for PoW, lock step round-by-round
Convergence Opportunities	Nakamoto Proof-of-Work	PSS'17	Tighter security for PoW, Δ-synchronous
Forkable Strings	Ouroboros Proof-of-Stake	KRDO'17, DGKR'18, BGK+'18, KQR'20	Tight security in the lock step round-by-round for PoS
Pivots	Sleepy/SnowWhite Proof-of-Stake	PS'17, BPS'16	Δ-synchronous
Blocktree Partitioning	**Any** longest chain protocol!	DKT'20	**Tight** security bound for PoW, PoS, PoSpace

Everything is a Race and Nakamoto Always Wins

https://arxiv.org/abs/2005.10484

Appendix: Branching Random Walk



Lemma: For $m \geq 1$, $P(D_i(t) \geq m) \leq \left(\frac{e\lambda_a t}{m}\right)^m$

Proof: (Branching Random Walk)

Appendix: Is private attack the worst attack?

In a statistical sense…

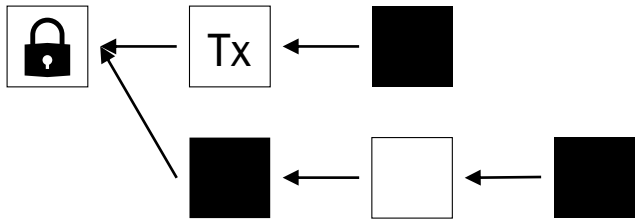
YES!

If the private attack fails, the protocol is secure with high probability.

In a deterministic sense…

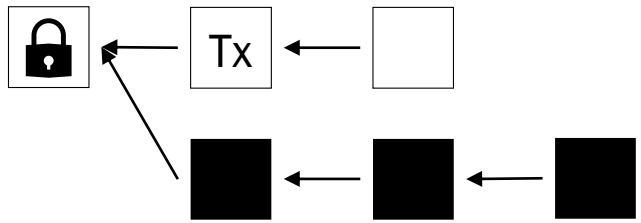
Only for the Proof-of-Work with network delay $\Delta = 0$.

Arbitrary Attack



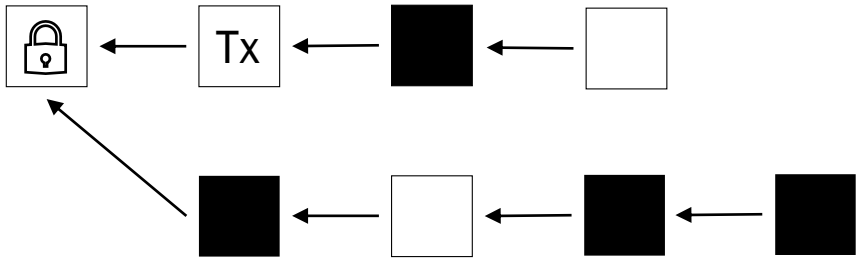
One honest block per level

Private Attack



Appendix: A Different Attack

k deep confirmation rule
(k =3)



Balance Attack succeeds!

Safety: Once Tx is k-deep, Tx stays as part of the longest chain.

Liveness: Longest chain contains honest blocks.